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# Determination of Hamiltonian Polygonal Paths of Assembly Graph for $FTTM_5$ Using Omega Algebra

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**Abstract**
Fuzzy Topographic Topological Mapping is a system for figuring out the location of epileptic foci. The FTTM has four main components that are homeomorphic to each other. They are magnetic contour plane (MC), base magnetic plane (BM), fuzzy magnetic field (FM) and topographic magnetic field (TM). The purpose of this study is to determine Hamiltonian polygonal paths of assembly graph of $FTTM_5$ using omega-algebra. All the concepts of Hamiltonian paths that are available in literature are reviewed in this project. The definition of polygonal path, assembly graph and omega algebra are introduced in this study. The concept of omega algebra to determine the Hamiltonian polygonal paths for $FTTM_5$ are studied and highlighted. In addition, the set of Hamiltonian polygonal paths are generated using programming software. Thus, the Hamiltonian Polygonal paths of assembly graph for $FTTM_5$ using omega algebra are obtained by a new algorithm. Hence, the visualization of the generated Hamiltonian paths is possible.
**Keywords:** Fuzzy topographic topological mapping, Hamiltonian paths; assembly graph; omega algebra

## Introduction

The concept of Fuzzy Topographic Topological Mapping (FTTM) was first introduced by UTM's Fuzzy Research Group in 1999 (Tahir et al., 2000). It has four main components that are homeomorphic to each other which implies that there are connections between any two components in this fuzzy graph. A sequence of $FTTM_n$ is a combination of *n* terms of FTTM. For example, $FTTM_5$ means five terms of FTTM are connect.

In this study, Hamiltonian polygonal paths of assembly graph for $FTTM_5$ are determined. An assembly graph is defined as a finite connected graph with number of endpoints are always even. Moreover, a polygonal path of assembly graph is when two immediate edges are neighbours with respect to their common vertex (Burn et al, 2013). As we know, a Hamiltonian path occurs when every vertex is visited exactly once.The set of Hamiltonian paths can be determined by using the concept of omega algebra. The assembly graph can be transformed into omega algebra.

Therefore, the main purpose of this study is to determine the Hamiltonian polygonal paths of assembly graph for $FTTM_5$ using omega algebra**.** The set of Hamiltonian polygonal paths are not trivial to represent them. Hence, C++ and Python programming languages are employed for the purpose.

## Assembly Graph of $FTTM_n$

The graph of $FTTM_n$ contains many subgraphs including assembly graphs. Hence, a concept called maximal assembly graph was introduced (Ahmad et al., 2019). The maximal assembly subgraph for $FTTM_n$ is a graph where the edges from the first and the last terms of FTTM namely $FTTM_1$ and $FTTM_n$ are removed. The term of a maximal assembly graph of $FTTM_n$ means four-valent subgraph. Then, the definition and theorems for graph of $FTTM_n$ are stated as follow:

**Definition 1 [2]**
The presentation of maximal assembly graph of $FTTM_n$: $\Gamma_{FTTM_n} = FTTM_n - [E(FTTM_1) \cup E(FTTM_n)]$ for n ≥ 3 such that $|\Gamma_{FTTM_n}|$ is the number of 4-valent vertices.
Unless otherwise stated, the maximal assembly subgraph of $FTTM_n$ is used in this work as an assembly

graph of $FTTM_n$. Ahmad, T. *et al.* (2019) stated few theorems of an assembly graph of $FTTM_n$ for n ≥ 3 as follow:

**Theorem 1 [2]** The $FTTM_3$ consists of an assembly subgraph.

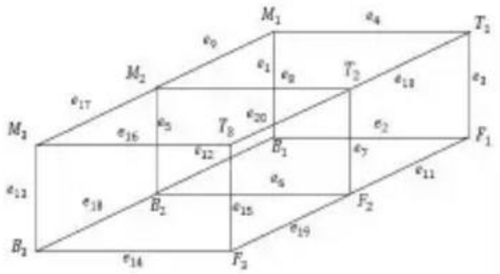

**Figure 1** **Geometrical features of $FTTM_3$**

The four vertices that have valency of four are $M_2, B_2, F_2, T_2$ in particular,
valency ($M_2$) = | $\{e_5, e_8, e_9, e_{17}\}$ | = valency ($B_2$) = | $\{e_5, e_6, e_{12}, e_{18}\}$ | = valency ($F_2$) = | $\{e_6, e_7, e_{11}, e_{19}\}$ | = valency ($T_2$) = | $\{e_7, e_8, e_{10}, e_{20}\}$ | = 4

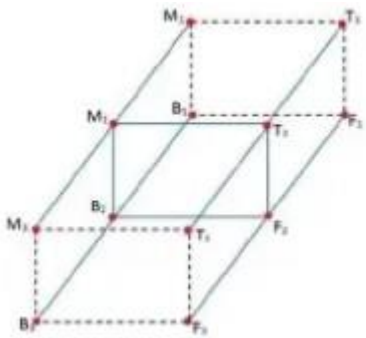

**Figure 2** **Assembly graph of $FTTM_3$**

Then, $M_1, B_1, F_1, T_1, M_3, B_3, F_3$ *and* $T_3$ have valency of one as stated:
valency ($M_1$) = | $\{e_9\}$ | = valency ($B_1$) = | $\{e_{12}\}$ | = valency ($F_1$) = | $\{e_{11}\}$ | = valency ($T_1$) = | $\{e_{10}\}$ | = valency ($M_3$) = | $\{e_{17}\}$ | = valency ($B_3$) = | $\{e_{18}\}$ | = valency ($F_3$) = | $\{e_{19}\}$ | = valency ($T_3$) = | $\{e_{20}\}$ | = 1

Therefore, $FTTM_3$ consists of an assembly subgraph with $|\Gamma_{FTTM_3}|$ = 4

### Hamiltonian Paths in an Assembly Graph of $FTTM_n$

In the preceding section, we review that assembly graph exists in any sequence of $FTTM_n$ for n ≥ 3. Any assembly graph of type of $FTTM_n$ contains Hamiltonian polygonal paths as well. As mentioned earlier, a set of Hamiltonian polygonal paths is denoted as $\gamma = \{\gamma_1, \dots, \gamma_s\}$. There are several theorems that can be used to determine the Hamiltonian polygonal paths of assembly graph for n ≥ 3.

**Theorem 2 [2]** The $\Gamma_{FTTM_3}$ consists of a set of Hamiltonian polygonal paths.

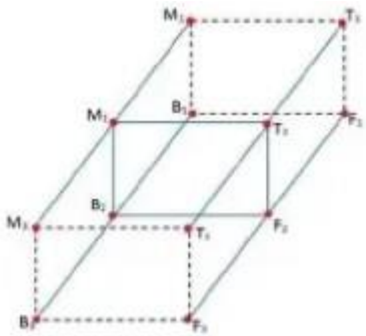


**Figure 3** **The Hamiltonian polygonal path of $FTTM_3$**

From Figure 3, the set of Hamiltonian polygonal paths are:

$\gamma_1 = \{M_2, e_5, B_2, e_6, F_2, e_7, T_2\}$ $\gamma_5 = \{F_2, e_7, T_2, e_8, M_2, e_5, B_2\}$

$\gamma_2 = \{M_2, e_8, T_2, e_7, F_2, e_6, B_2\}$ $\gamma_6 = \{F_2, e_6, B_2, e_5, M_2, e_8, T_2\}$

$\gamma_3 = \{B_2, e_5, M_2, e_8, T_2, e_7, F_2\}$ $\gamma_7 = \{T_2, e_8, M_2, e_5, B_2, e_6, F_2\}$

$\gamma_4 = \{B_2, e_6, F_2, e_7, T_2, e_6, M_2\}$ $\gamma_8 = \{T_2, e_7, F_2, e_6, B_2, e_5, M_2\}$

Therefore, the $\Gamma_{FTTM_3}$ consists of eight Hamiltonian polygonal paths and $\|\gamma\| = 24$

The total number of sets of Hamiltonian polygonal paths in an assembly graph of $FTTM_n$ that was determined by Mahamud & Ahmad (2017). Hence, this Table 1 summarizes the number of polygonal paths in an assembly graph of $FTTM_n$ for $n = 3,4,5, \dots, 10$.

**Table 1:** Hamiltonian polygonal paths in assembly graph of $FTTM_n$

| $FTTM_n$ | No. of vertices | No. of 4-valent vertices | Hamiltonian Polygonal Paths |
|---|---|---|---|
| $FTTM_3$ | 12 | 4 | 8 |
| $FTTM_4$ | 16 | 8 | 144 |
| $FTTM_5$ | 20 | 12 | 1,168 |
| $FTTM_6$ | 24 | 16 | 8,032 |
| $FTTM_7$ | 28 | 20 | 49,312 |
| $FTTM_8$ | 32 | 24 | 281,248 |
| $FTTM_9$ | 36 | 28 | 1,523,920 |
| $FTTM_{10}$ | 40 | 32 | 7,953,408 |

**Omega Algebra**

The definition of omega algebra is a procedure to form a new element from one or more input entities. Given a set *M*, the *N*-ary algebraic operation on the set *M* is expressed as $\omega_n : M_1 \times M_2 \times \dots \times M_n \to M$ Thus, to sum up, the n-ary of arity of a function or operation is the number of operands of the function. For some set *M,* ω is an operation and *n* are its arity, $\omega_n : M^n \to M$. Therefore, omega algebra (Ω – Algebra) is a set by means of a defined system of operations Ω namely, Ω – Algebra = $\{\omega_k \mid k = \{0,1, \dots, n\}\}$ which is defined on one primary set namely one-sorted algebraic system (Plotkin et al., 1992). The concept of omega algebra can be viewed as a system of mapping such that:

$\omega_2 : M \times M$
$\omega_3 : M \times M \times M$

$\vdots$

$$\omega_n : \underbrace{M \times M \times \ldots \times M}_{n \text{ times}}$$

In addition, the * function of the omega algebra is employed to depict the direct and indirect catalytic relations in secondary system of PWR (Zainab, et al. 2018). The operations based on length and number of paths is given in Table 2.

**Table 2**: Performance of operations based on length of the paths with its number of paths.

| Operations | N times = Length | Number of Paths |
|---|---|---|
| Binary | 1 (*2) | 21 |
| Ternary | 2 (*3) | 50 |
| Quaternary | 3 (*4) | 93 |
| Quinary | 4 (*5) | 106 |
| Senary | 5 (*6) | 47 |
| Septenary | 6 (*7) | 0 |

## Theoretical Results

*Determination of Hamiltonian Paths of Assembly Graph for* $FTTM_5$

$FTTM_5$ consists of an assembly subgraph with $|\Gamma_{FTTM_5}|$ = 12

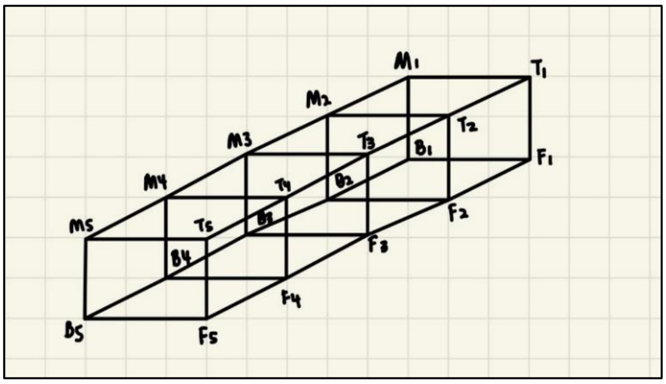


**Figure 4** **Geometrical features of $FTTM_5$**

In this study, we have been introduced to a new concept which is called maximal graph. It is a graph where the edges from first and last term of $FTTM_n$ are removed. The, assembly graph of $FTTM_5$ in Figure 5 is obtained with 12 valent vertices that have valency of four are $M_2, B_2, F_2, T_2, M_3, B_3, F_3, T_3, M_4, B_4, F_4$ and $T_4$. For instance,

$$\begin{aligned} valency(M_2) = &|\{e_5,e_8,e_9,e_{17}\}| = valency(B_2) = |\{e_5,e_6,e_{10},e_{18}\}| = valency(F_2) \\ &= |\{e_6,e_7,e_{11},e_{19}\}| = valency(T_2) = |\{e_7,e_8,e_{12},e_{20}\}| = valency(M_3) \\ &= |\{e_{13},e_{16},e_{17},e_{25}\}| = valency(B_3) = |\{e_{13},e_{14},e_{18},e_{26}\}| = valency(F_3) \\ &= |\{e_{14},e_{15},e_{19},e_{27}\}| = valency(T_3) = |\{e_{15},e_{16},e_{20},e_{28}\}| = valency(M_4) \\ &= |\{e_{21},e_{24},e_{25},e_{33}\}| = valency(B_4) = |\{e_{21},e_{22},e_{26},e_{34}\}| = valency(F_4) \\ &= |\{e_{22},e_{23},e_{27},e_{35}\}| = valency(T_4) = |\{e_{23},e_{24},e_{28},e_{36}\}| = 12 \end{aligned}$$

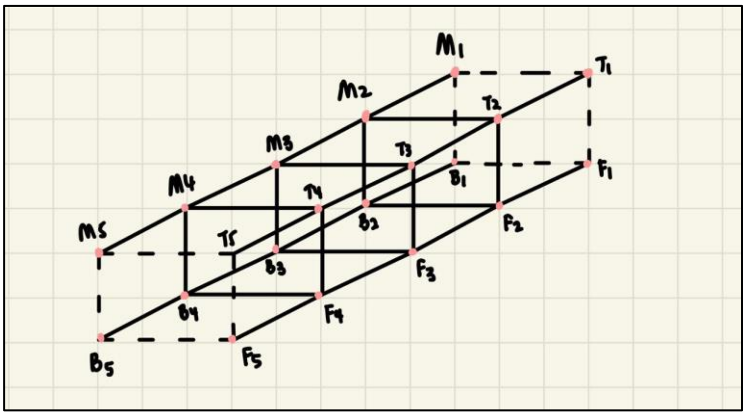


**Figure 5** **Assembly Graph of $FTTM_5$**

Therefore, the $FTTM_5$ consists of an assembly subgraph with $|\Gamma_{FTTM_5}|$ = 12

■

$FTTM_5$ consists of a set of Hamiltonian polygonal paths.

Figure 6 exhibits the Hamiltonian polygonal paths of assembly graph for $FTTM_5$. As mentioned above, edges near endpoints are not Hamiltonian polygonal pathways, and endpoints are vertices with valency of 1. Therefore, the edges and four valent vertices that are enclosed in Hamiltonian polygonal paths are stated as follow:

$$\{M_2,B_2,F_2,T_2,M_3,B_3,F_3,T_3,M_4,B_4,F_4,T_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13},e_{14},e_{15},e_{16},e_{17},$$
$$e_{18},e_{19},e_{20},e_{21},e_{22},e_{23},e_{24},e_{25},e_{26},e_{27},e_{28}\}$$

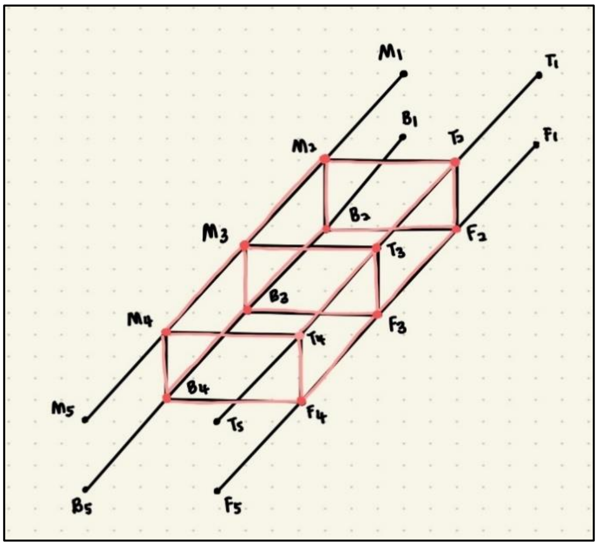


**Figure 6** **The Hamiltonian polygonal paths of assembly graph of $FTTM_5$**

From Figure 6, $FTTM_5$ attains 1,168 Hamiltonian polygonal paths and the set of paths are determined by using C++ programming language. There is one of Hamiltonian polygonal paths is:

$$\gamma_1 = \{M_2, e_8, T_2, e_7, F_2, e_6, B_2, e_{10}, B_3, e_{13}, M_3, e_{16}, T_3, e_{15}, F_3, e_{14}, B_3, e_{26}, B_{4,} e_{21}, M_{4,} e_{24}, T_{4,} e_{23}, F_4\}$$

The $\Gamma_{FTTM_5}$ consists of 1,168 Hamiltonian polygonal paths.

■

*Determination of Hamiltonian Paths of Assembly Graph for $FTTM_5$ using omega algebra*

Let $\Omega_{FTTM_5}$ is the omega algebra of $FTTM_5$.

Consider $V_{G_{FTTM_5}} = \Gamma_{FTTM_5}$. The path of Hamiltonian paths of $\Omega_{FTTM_5}$ can be viewed as system mapping such that:

$$\omega_{12}: \underbrace{M \times M \times M \times M \times M \times M \times M \times M \times M \times M \times M \times M}_{n \text{ times = length}} \rightarrow M$$

Figure 7 presents a path with length 11 is stated as follow:

- Duodenary operation

  $\omega_{12}: *_{12} : V \times V \times V \times V \times V \times V \times V \times V \times V \times V \times V \times V \rightarrow V$

  such that $v_i$ , $v_j$, $v_{k,}$ $v_l$, $v_m$, $v_n$, $v_0$, $v_p$, $v_{q,}$ $v_r$, $v_s$, $v_t \in V$ and $v_i *_{12} v_t = v_t \in V$

  Through $v_j, v_k$, $v_l$, $v_m$, $v_n$, $v_0$, $v_p$, $v_{q,}$ $v_r$ and $v_s$. (Twelve vertices with length 11)

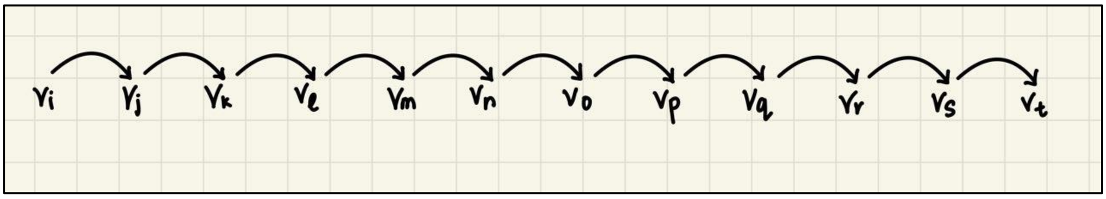


**Figure 7** **A path with length 11**

*Implementation*

Algorithm and Flowchart

Writing and creating an algorithm can be such a complex task An algorithm can be defined as a procedure or set of rules that must be followed when performing calculations or other problem-solving tasks. An algorithm is a set of rules/instructions that govern how an operation should be completed to get the desired results.

Moreover, a flowchart is a diagram that depicts an algorithm. As we know, programmers frequently utilise the flowchart as a program-planning tool in order to deal with the problem given. It employs symbols that are linked together to represent the flow of information and processing. "Flowcharting" refers to the process of creating a flowchart for an algorithm. It provides a description of the important logical steps of the process The process to determine the possible paths present in a graph is shown as follows:

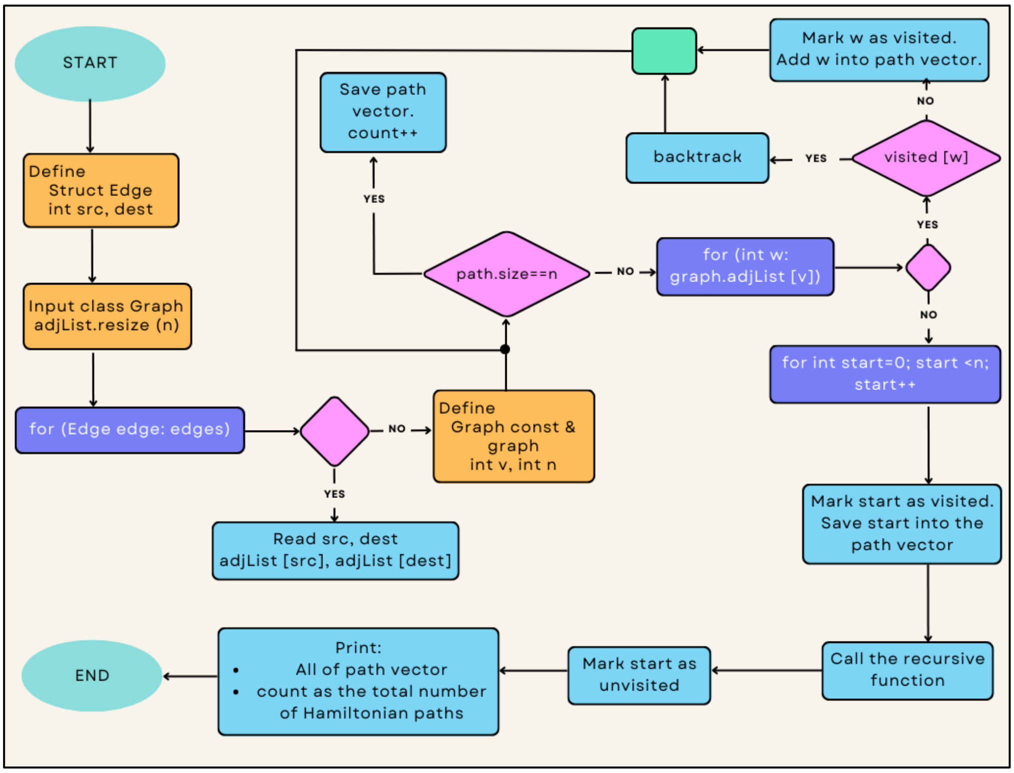


**Figure 8** **Flowchart of possible length of Hamiltonian Paths**

In addition, the main idea is to find the possible paths of $FTTM_5$ using backtracking techniques. This technique requires every edge starting from an unvisited vertex to be searched. It is because a Hamiltonian path visits every vertex precisely once.

First and foremost, we construct a class of graphs. Then, we create a vector of vectors to represent an adjacency list of edges to the undirected graph. After that, we utilize the array to hold the vertices traversed by the current path. If all of the vertices are visited, then the graph has a Hamiltonian path. If not, we use backtracking by unmarking the adjacent vertex as visited for every adjacent vertex. Lastly, we find the Hamiltonian paths with assigned the starting point for every node. Retrace the steps with different beginning vertices.

There are new algorithms that are developed and coded to visualize the possible paths present in a graph. The graph representation nodes labelled from 0 to 11 is shown in Figure 9.

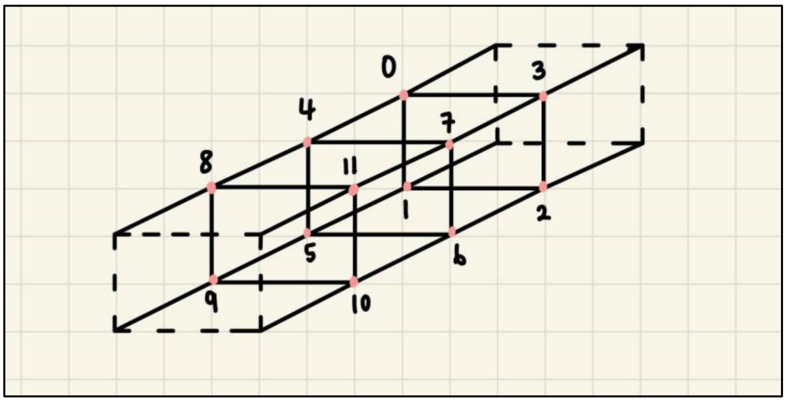


**Figure 9** **Graph representation of $FTTM_5$**

Visualization of Hamiltonian Paths

Multiple approaches can be used to visualize Hamiltonian Paths as an undirected graph. Visualization can be defined as representation of data analysis using charts, graphs, maps, and other tools. On the other hand, there are various graph visualizations tools available to aid in the visualization of Hamiltonian paths. They frequently include highlighting pathways, labelling vertices, and automatically generating graph layouts. For instance, NetworkX, Graphviz, and Gephi are some prominent graph visualization tools.

We used NetworkX as our graph visualization tool for this project and developed its coded (Appendix B). NetworkX constitutes a Python package for creating, manipulating, and studying complex networks' structure, dynamics, and functions. There are several procedures that need to be considered to visualize Hamiltonian paths and to animate the direction of the paths. The steps are given as follow:

1. Need to Install NetworkX library in Python environment by using:
    - pip install networkx
2. Import networkx, matplotlib and any additional libraries required in python script.
3. Create the Graph and define the graph using NetworkX. Use the function given to create the graph based on the specific graph structure.
    - add_node ( )
    - add_edge ( )
4. Specify the new labels for the nodes using function new_labels { }
5. Find the Hamiltonian Paths by read from text file which contains the coded from C++ programming language. In order to access the text file and read its contents, use the Python open ( ) function. It is possible to perform an iteration over the lines and split each line to acquire the path's individual nodes. For additional processing or visualization, relocate the Hamiltonian routes in a list or any other acceptable data structure.
6. Visualize Hamiltonian paths in circular layout which the placements of the nodes in a circular configuration are determined automatically using the function given:
    - circular_layout ( )
7. Set up the animation have few steps following by the functions to generate the animation of the Hamiltonian Paths. The steps are given as stated:
    i. Use function figsize ( ) to set the size of the figure.
    ii. Use function animate ( ) and define it to call each frame of the animation.
    iii. Use function draw_networkx ( ) to visualize the graph and Hamiltonian paths are emphasized using function draw_networkx_edges ( ).

iv. Use function draw_networkx ( ) to visualize the graph and Hamiltonian paths are emphasized using function draw_networkx_edges ( ). In order to know the cycle of the path, we define a list colour to cycle through with function given:

- colors = cycle ([ 'red', 'green', 'blue', 'orange', 'magenta'])
- color_cycle = cycle (colors)

Then, the function where node_color = next (color_cycle) will use to get the next color from the cyle. Note that we need to import cycle from itertools.

v. Animate ( ) method is called for each frame of the animation by the animation.FuncAnimation ( ). The interval parameter (in milliseconds) allows to adjust the time between frames.

8. Display the animation using plt.show ( ) function.
9. The animation of the paths can save as an image by saving the current frame using the function plt.savefig( ) and to make it easier for user to know the saved images, we will create the directory to save images by the function of save_dir and os.makedirs ( ). Make sure to import os module before used it.

As we know that the total of Hamiltonian paths of assembly graph for $FTTM_5$ is very large Visualizing the path is very challenging since it may be impossible to locate all Hamiltonian paths manually. Therefore, this animation of Hamiltonian paths was created to present the path in a more efficient and practical manner. Figure 10 shows an example of the image of possible lengths of a Hamiltonian Path for $FTTM_5$.

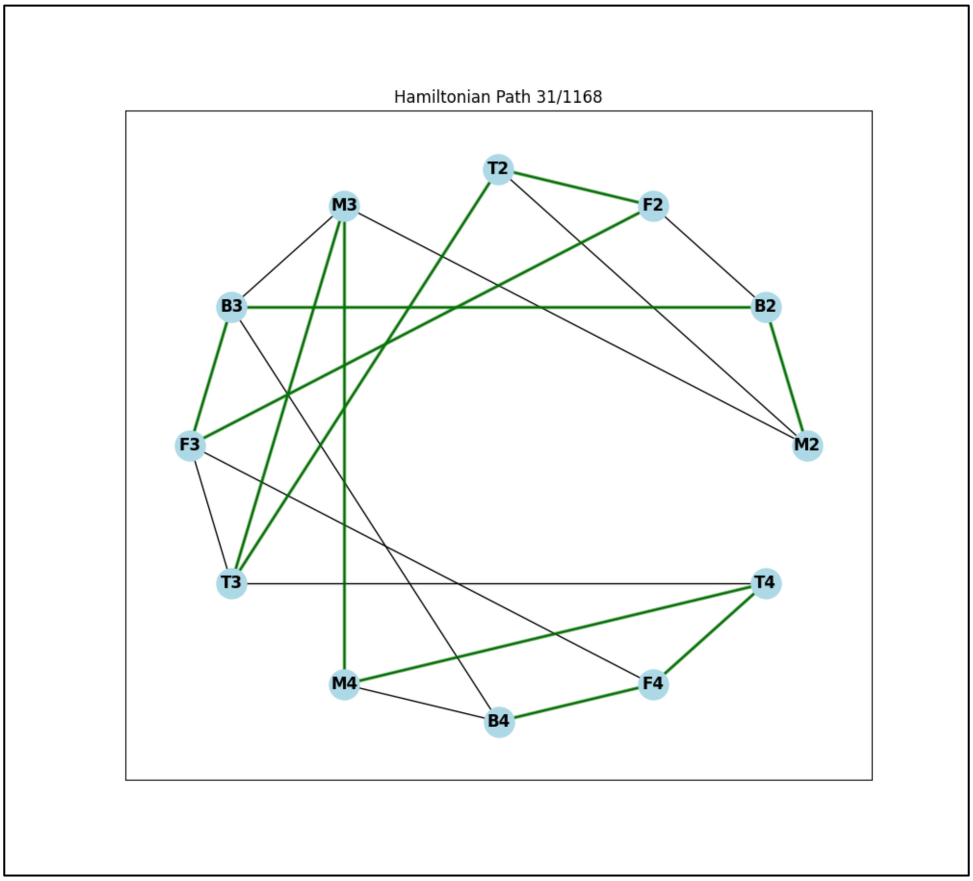


**Figure 10** **A Hamiltonian Path of $FTTM_5$.**

**Conclusion**

In this paper, the Hamiltonian polygonal paths of assembly graph for $FTTM_5$ are determined by using the concept of omega algebra. This concept can generate all possible paths and lengths of Hamiltonian polygonal paths. In addition, the relationship between Hamiltonian paths of $G_{FTTM_5}$ and omega, Ω algebra is shown in this project. The results for this project were obtained using some programming languages. The programming languages include C++ programming and Python. The main objectives are to generate the set of paths and to animate the paths using the programming languages stated. Therefore, in future research, more research can be conducted in this topic, such as on how to find the possible paths of Hamiltonian paths in a different programming language. Even though, numerous researchers have established and refined the concept of assembly graphs of $FTTM_n$, however, to obtain the possible paths of Hamiltonian paths of other $FTTM_n$ remains a challenge.

**Acknowledgement**

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**References**

[1] Ahmad, T. *et al.* (2005) *Homeomorphisms of fuzzy topographic topological mapping(fttm)*, *Research Gate*. Available at: https://www.researchgate.net/profile/Tahir-Ahmad-8/publication/26605437_Homeomorphisms_of_Fuzzy_Topographic_Topological_MappingFTTM/links/54827f8d0cf25dbd59ea96c8/Homeomorphisms-of-Fuzzy-Topographic-Topological-MappingFTTM.pdf (Accessed: January 25, 2023).

[2] Ahmad, T. *et al.* (2019) "Hamiltonian polygonal path in assembly graph of FTTM," *PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON MATHEMATICAL SCIENCES AND TECHNOLOGY 2018 (MATHTECH2018): Innovative Technologies for Mathematics & Mathematics for Technological Innovation* [Preprint]. Available at: https://doi.org/10.1063/1.5136356.

[3] Burns, J. *et al.* (2013) "Four-regular graphs with rigid vertices associated to DNA recombination," *Discrete Applied Mathematics*, 161(10-11), pp. 1378–1394. Available at: https://doi.org/10.1016/j.dam.2013.01.003.

[4] Developers, N. (2014). *NetworkX documentation*. NetworkX. https://networkx.org/

[5] Zainab Binti Mahamud. (2019) *Length and Order of Fuzzy Autocatalytic Set of a Pressurized Water Reactor* [Preprint]

[6] Zainab, M., Tahir, A. and Siaw Yee, L. (2018) *Semigroup Transformation for Fuzzy Autocatalytic Set Catalytic Relation Models*. Johor.