

## MHD NANOFUID FLOW IN PRESENCE OF HEAT SOURCE/SINK CONSIDERING SHAPE EFFECTS OF NANOPARTICLES

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### Abstract

The study investigates the magnetohydrodynamic (MHD) flow of nanofluids with the presence of a heat source/sink, considering the shape effects of nanoparticles. Nanofluids, consisting of nanoparticles suspended in a base fluid, exhibit enhanced thermal properties compared to conventional fluids. This research focuses on understanding how different nanoparticle shapes influence the thermal conductivity and heat transfer efficiency of MHD nanofluids. The presence of a magnetic field is considered to control the heat transfer characteristics in these fluids, which has significant applications in various industries such as aeronautics, radiation therapy, and MHD generators. The significance of this study lies in its potential to improve thermal management in various engineering applications by optimizing the shape of nanoparticles for better heat transfer efficiency. The findings are anticipated to provide practical guidelines for designing and optimizing systems where heat transfer efficiency is critical. The study's outcomes could lead to advancements in industries such as medical technology, transportation, and solar energy devices, where enhanced thermal conductivity and stability of nanofluids are highly beneficial.

Keywords: magnetohydrodynamic; nanofluids; heat source/sink

### Introduction

Fluids often act as heat carriers in a heat transfer process. Fluids are used to transport and store thermal energy in numerous industrial applications that require heating systems such as food and beverage processing, wood manufacturing and heat exchanger. In order to ensure the efficiency of heat transfer equipment, the characteristics of heat transfer fluids are considered. For instance, fluid with low viscosity will not exhibit much resistance to the heat transfer. Heat transfer fluid with high thermal conductivity will have a higher rate of heat transfer as molecules will move more quickly at higher temperatures.

Nanofluid is a fluidic system comprised of a colloidal suspension of nanoparticles, which are particles of nanometer size. These nanoparticles are suspended in a base fluid that possesses low thermal conductivity, such as water, ethylene glycol, and oils. The commonly employed nanoparticles in nanofluids are metal or metal oxide nanoparticles, such as copper and alumina. The incorporation of nanoparticles into conventional heat transfer liquids results in an enhancement of the fluid's thermal conductivity and heat transfer characteristics. It is recommended to employ nanoparticles with a large surface area in order to improve the heat transfer capabilities, as the heat transfer occurs at the surface of the nanoparticle. Nanofluids are anticipated to exhibit superior characteristics in comparison to traditional heat transfer fluids, and they do not cause blockages in flow passages. Therefore, these nanoparticles and base fluid systems are renowned for their exceptional heat transfer properties, rendering them valuable in various industry sectors.

The occurrence of a magnetic field has the capability to induce electric currents within a fluid that is capable of conducting electricity. In the field of magnetohydrodynamics (MHD), this phenomenon is exploited. By virtue of the applied magnetic field, the thermal absorption within

electrically conductive fluid flows is influenced, thereby allowing for control over the desired attributes of the final product. Consequently, this circumstance has given rise to significant applications in various industries, such as radiation therapy, aeronautics, and MHD generators. The presence of a magnetic field can potentially exert an influence over the regulation of heat transfer in the boundary layer flow occurring over a surface undergoing stretching. It is highly efficacious in shaping the thermal properties of nanofluids and discerning their heat transfer characteristics. Hence, it is highly recommended that the incorporation of magnetic field effects be considered in heating industries as a means to augment the efficacy of heat transfer operations (Khashi'ie et al., 2022).

Convection is a process of heat transfer that happens when molecules in fluids move. It only happens in liquids and gases; it doesn't happen in solids because there are no moving particles. Convection usually happens as the first method of heat transfer from an object to the fluid, and then it leads to bulk heat transfer because of the motion of the fluid. When it comes to the release of energy, there are two types of convection that can be seen: free convection and forced convection. Free convection, also called natural convection, is a flow pattern where a liquid moves because of its own density differences between different parts of the fluid. On the other hand, forced convection is a technique where a force from outside the system causes fluid motion. Mixed convection in heat transmission refers to the mixture of forced and free convection. The main goal of this study is to investigate basic questions about the effects of nanoparticle form on MHD nanofluid flow behaviour. The goal of this research is to promote more sustainable and effective thermal management technologies by investigating these links. It is predicted that the results would have practical ramifications for a broad spectrum of engineering applications, offering direction for the design and optimisation of systems where heat source form plays a crucial role in determining heat transfer efficiency.

**Problem formulation**

Consider two-dimensional boundary layer flow of a nanofluid through the surface stretched along x-axis. The linear velocity is considered and is defined as  $u_w(x) = ax$  where  $a$  is a constant. Furthermore, it is supposed that  $B_0$  magnetic field is imposed normally to the stretching sheet. Figure 3.1 displays the coordinate system of the considered physical model.

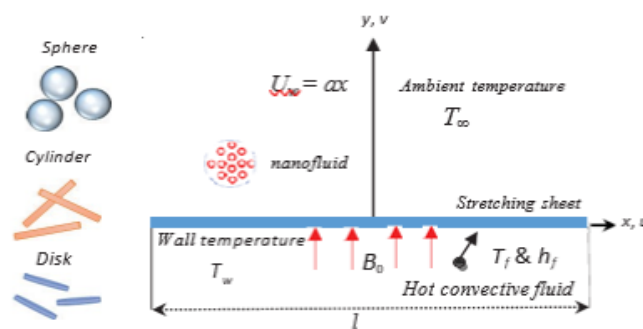


Figure 3. 1 Physical model and coordinate

The governing equations of the considered problem are as follows:

$$\frac{du}{dx} + \frac{dv}{dy} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} \beta_0^2 u}{\rho_{nf}} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{q}{(\rho c_p)_{nf}} (T - T_\infty) \tag{3}$$

subject to the following boundary conditions

$$u(x, 0) = U_w(x) = ax, v(x, 0) = 0, -k \frac{\partial T}{\partial y}(x, 0) = h_f [T_f - T(x, 0)], \text{ at } (y = 0) \tag{4}$$

$$u(x, \infty) = 0, v(x, \infty) = 0, T(x, \infty) = T_\infty \text{ at } (y \rightarrow \infty)$$

where  $u$  and  $v$  are the velocity components along  $x$  and  $y$  directions, respectively.  $T$  is the temperature of nanofluid,  $T_\infty$  is the free stream temperature,  $\rho_{nf}$  ( $\rho_{cp}$ ) $_{nf}$  and  $\sigma_{nf}$  are density, heat capacitance and electrical conductivity of the nanofluid, respectively, and are defined as follows:

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_s \tag{5}$$

$$(\rho c_p)_{nf} = (1 - \varphi)(\rho c_p)_f + \varphi(\rho c_p)_s \tag{6}$$

$$\sigma_{nf} = \left( 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\varphi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)} \right) \sigma_f \tag{7}$$

The dynamic viscosity of the nanofluid  $\mu_{nf}$  for spherical nanoparticle is given as follows:

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \tag{8}$$

The dynamic viscosity of the nanofluid  $\mu_{nf}$  for non-spherical nanoparticle shapes (Table 1) is given as follows (Ellahi et al, 2016):

$$\mu_{nf} = \mu_f (1 + A\varphi + B\varphi^2) \tag{9}$$

Here, the constants  $A$  and  $B$  depend on the nanoparticle shape, and their value is as mentioned in Table 3.1. The nanofluid thermal conductivity is defined as follows (Gireesha et al, 2020):

$$\frac{k_{nf}}{k_f} = \frac{k_s + (m - 1)k_f - (m - 1)\varphi(k_f - k_s)}{k_s + (m - 1)k_f + \varphi(k_f - k_s)} \tag{10}$$

where  $m$  is the empirical shape factor ( $m = 3 - \psi$ ) where  $\psi$  is the sphericity and its values for nanoparticles of different shapes are noted in Figure 3.2.

Let  $\psi$  is stream function with  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$ . Consider the following transformations:

$$\eta = \sqrt{\frac{a}{\nu_f}} x f(\eta), \psi = \sqrt{a \nu_f} y, \theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{11}$$

On applying the similarity constraints, the momentum and energy equations take the following forms:

$$f''' + \frac{\mu_f}{\mu_{nf}} \left\{ \frac{\rho_{nf}}{\rho_f} (ff'' - f'^2) - \frac{\sigma_{nf}}{\sigma_f} Mf' \right\} = 0, \tag{12}$$

$$\frac{k_{nf}}{k_f} \theta'' + \text{Pr} \left\{ \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} f \theta' + \lambda \theta \right\} = 0 \tag{13}$$

Table 3.1 Value A and B for different shaped nanoparticle

Model	brick shape	cylinder shape	platelets shape	disk shape
A	1.9	13.5	37.1	14.6
B	471.4	904.4	612.6	123.3






Nanoparticle shapes	Sphere	Bricks	Cylinder	Platelets	Disk
Shape structure					
Sphericity $\psi$	1	0.81	0.62	0.52	0.36

Figure 3. 2 Structure and sphericity,  $\psi$  of different nanoparticle shapes

The resultant boundary conditions are as follows:

$$\begin{aligned} f(0) = 0, f'(0) = 1, \theta'(0) = -Bi[1 - \theta(0)] \\ f(\infty) = 0, \theta(\infty) = 0 \end{aligned} \tag{14}$$

**Results and discussion**

The transformed equations is solved by using MATLAB bvp4c solver. The numerical results obtained are presented graphically with parameter values specified in each figure. A comparison between the present results and the previous findings of Xu and Lee (2013), Hayat et.al (2020),

Mabood et al (2020) and Hamza et. al (2021), concerning the skin friction and Nusselt number is presented in the table below:

Table 4.1 Comparison table for skin friction for various value of M when  $\phi = 0$  and Pr = 0

M	Xu and Lee (2013)	Hayat et.al (2020)	Mabood et. Al (2020)	Hamza et. al (2021)	Present study
1	-1.41421	-1.41421	-1.41421	-1.41421	-1.4142113
5	-2.4994	-2.44948	-2.44948	-2.44948	-2.4494897
10	-3.3166	-3.31662	-3.31662	-3.31662	-3.3166248
50	-7.1414	-7.14142	-7.14142	-7.14142	-7.1414284
100	-10.0498	-10.04987	-10.0498	-10.0498	-10.04987

Table 4.2 Comparison table for Nusselt number for various value of Pr when M = 0 and  $\phi = 0$

Pr	Xu and Lee (2013)	Hayat et.al (2020)	Mabood et. Al (2020)	Hamza et. Al (2021)	Present study
0.7	0.4539	0.45396	0.45048	0.454444	0.454426
2	0.9114	0.91139	0.91135	0.911352	0.911264
7	1.8594	1.85942	1.85940	1.859400	1.895041
20	3.3559	3.35386	3.35390	3.353901	3.352777

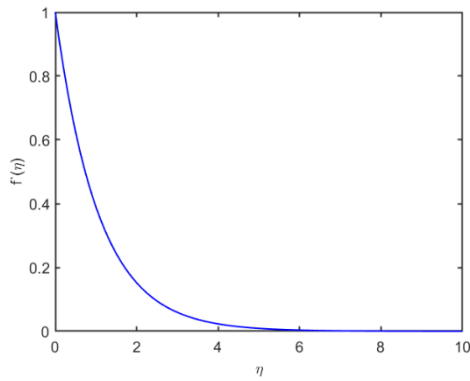


Figure 4.1(a)

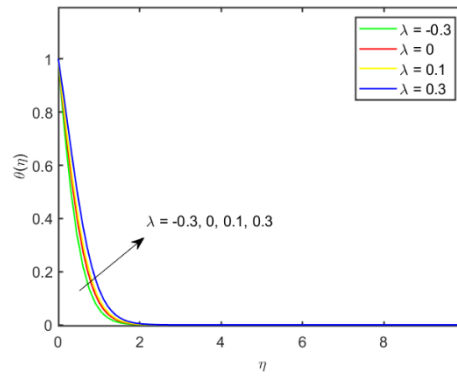


Figure 4.1(b)

Figure 4.1(a) and 4.1(b) shows the positive values of  $\lambda$  represent heat source meanwhile the negative values of  $\lambda$  represent heat sink. As the values of  $\lambda$  increases, it is shown that the temperature profiles also increase. Different  $\lambda$  values can indicate different rates of heat transfer, affecting the temperature distribution in the fluid.

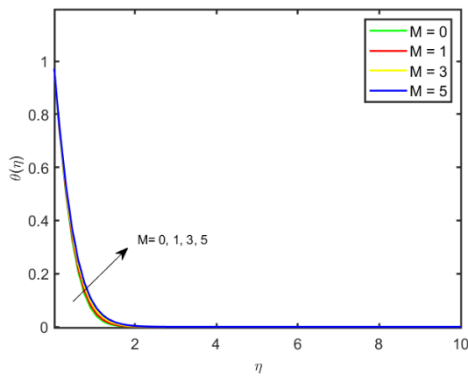


Figure 4.2(a)

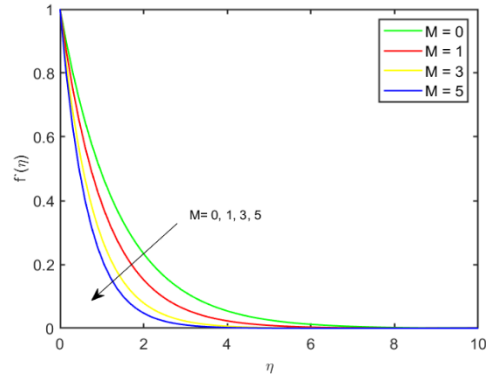


Figure 4.2(b)

Figure 4.2(a) and 4.2(b) presents the magnetic field,  $M$  on velocity and temperature profiles. From Figure 4.2(a) it can be examined that with the increase of magnetic field cause the fluid's temperature profile to decrease. On the other hand, the velocity distribution is seen to be increase as the magnetic field increase in Figure 4.2(b). This happen because the Lorentz force produced owing to magnetic field, which acts as a resistance to the flow of nanofluid. (Hamzah et.al, 2021).

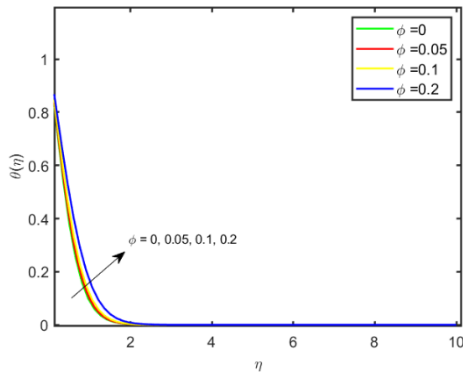


Figure 4.3(a)

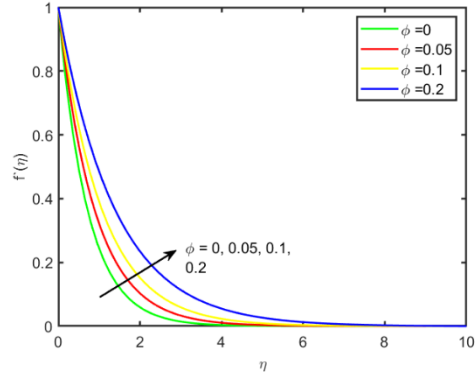


Figure 4.3(b)

Figure 4.3(a) and 4.3(b) presents the volume fraction,  $\phi$  on velocity and temperature profiles. Figure 4.3(a) presents the temperature profile for various values of nanoparticles volume fraction. An enhancement of the  $\phi$  in the range of 0 to 0.2 resulted in increasing the temperature profile. Figure 4.3(b) illustrates the velocity profile for different values of nanoparticles volume fraction. For the increasing volume of fraction, velocity profile also increases due to rise in concentration of nanoparticles.

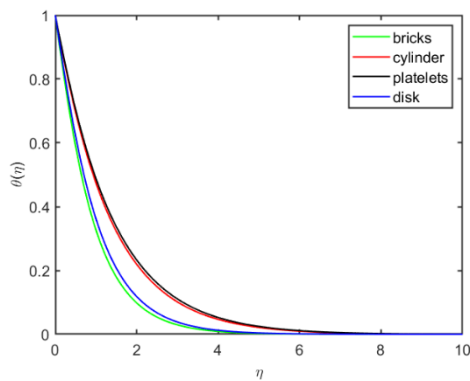


Figure 4.4(a)

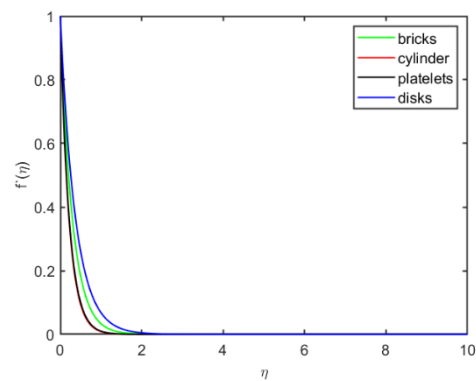


Figure 4.4(b)

Figure 4.4(a) and 4.4(b) show the temperature and velocity profile various value of shapes. Figure 4.4(a) shows that the temperature profile for bricks is lower than other shapes. The temperature profile for bricks is lower than platelets, cylinder and disks. Figure 4.4(b) shows that the velocity profile for disks is higher than other shapes. The velocity profile for disks is bigger than bricks, cylinder and platelets. This is because the inclusion of heat source generates energy while the heat sink absorbs energy. Hence, high temperature for heat source case and low temperature for heat sink case. (Hamzah et.al, 2021)

Table 4.3 Table for Nusselt number and number skin friction

parameter	$f'$	$\theta$
$\lambda$		
-0.3	-0.9552050	-1.8810488
0	-0.9552050	-1.4777128
0.1	-0.9552050	-1.3201933
0.3	-0.9552050	-0.9483515
M		
0	-0.7593444	-1.9084238
1	-0.9552050	-1.8816488
3	-1.2719185	-1.8377415
5	-1.5283382	-1.8041927
$\phi$		
0	-1.4159579	-1.2748580
0.05	-1.1424623	-1.2171700
0.1	-0.9552050	-1.1456881
0.2	-0.7520368	-0.9828813

**Conclusion**

This research investigates the magnetohydrodynamic (MHD) flow of nanofluids, focusing on the effects of nanoparticle shapes and the presence of heat sources or sinks. Using mathematical modeling and numerical solutions via MATLAB's BVP4C solver, the study found that different nanoparticle shapes significantly impact thermal conductivity and heat transfer efficiency. The presence of a magnetic field enhances control over heat transfer characteristics, and the effects of heat sources and sinks further modulate temperature profiles. These insights provide practical guidelines for optimizing thermal systems in various engineering applications, potentially benefiting industries like medical technology, transportation, and solar energy.

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