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Heat and Mass Transfer in Boundary Layer Flow Due to swimming of Motile Gyrotactic Microorganisms Towards a Flat Plate with Slip Velocity and Heat Generation

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Abstract

The purpose of this research project is to investigate the dynamics of heat and mass transfer in boundary layer flow induced by swimming motile gyrotactic microorganisms over a flat plate with slip velocity and heat generation. The study aims to gain a comprehensive understanding of the behavior of swimming microorganisms in the fluid settings. By applying similarity transformation, the governing nonlinear partial differential equation are transformed into a set of nonlinear ordinary differential equations. These equations are then solved numerically using MATLAB solver bvp4c, a method well-suited for boundary value problem. The analysis will investigate the effects of slip velocity and heat generation on the velocity, temperature, concentration, and movement of the microorganism, In addition, the research delves into physical parameters such skin friction coefficient, Nusselt number and Sherwood number. Results show that heat generation increases temperature and mass transfer while reducing the density of microorganism and fluid velocity due to lower viscosity. Slip velocity alters the shear stress and velocity gradient, improving system efficiency by reducing drag force and skin friction, and effects the concentration and microorganisms near the surface. The findings of this research have far- reaching implications for various practical applications. It provide valuable insights for optimizing practical applications in biotechnology, environmental engineering, and industrial processes.

Keywords: Heat and mass transfer; slip velocity; heat generation; motile gyrotactic microorganisms

Introduction

The transport of heat and mass are fundamental aspects of fluid flow and are central to numerous industrial and engineering applications. Heat transfer, in particular, has long been a subject of significant interest due to its critical role in various manufacturing processes such as those in the automotive, aerospace, chemical processing, oil and gas, and nuclear energy sectors. Heat transfer involves the movement of thermal energy from one object or system to another as a result of temperature differences. This process can occur through conduction, convection, or radiation. For instance, in a boiler, chemical energy from a fuel source like oil and gas is converted into heat by combustion. This heat is then transferred to water, converting it into steam. The design of such system aims to maximize heat transfer efficiency to ensure the most effective use of fuel. On the other hand, mass transfer refers to the movement of mass from one location to another, typically driven by concentration differences. A common example is found in internal combustion engines, where fuel injection represents a mass transfer process. Liquid fuel is injected into the combustion chamber, where it evaporates and mixes with air,facilitating the combustion process. While gyrotactic microorganisms are microorganisms that exhibit a characteristic behavior called gyrotaxis. Gyrotaxis refers to the tendency of these microorganisms to align and swim along the axis of a fluid flow or a gradient of certain environmental factors, such as light, temperature, or chemical concentration. Gyrotactic microorganisms can actively move and reorient themselves in response to fluid flow. This behavior can enhance the mixing of substances in the fluid, leading to increased heat and mass transfer.

The presence of gyrotactic microorganisms in fluid flow can significantly influence heat and mass transfer. Studies have used numerical approaches to investigate the boundary layer flow with heat and mass transfer in the presence of these microorganisms. Their findings indicated that heat transfer rates decrease with increasing magnetic and buoyancy effects, while mass transfer rates vary depending on the magnetic field and stretching parameters [1]. Further research highlighted the role of gyrotactic microorganisms in enhancing heat and mass transfer in convective flows containing nanoparticles and other substances. The movement of these microorganisms can induce microscopic convection in the fluid, improving the transport of heat, mass and nutrients. These findings have implications for various applications including enzyme biosensors, biotechnology, and microfluidic devices [2-3]. Studies demonstrated that the presence of microorganisms can enhance convection which leading to improved heat and and mass transfer characteristics [4-5]. Further investigation delved into the dynamics of bioconvective fluids, highlighting how the concentration of self-propelled microorganisms can lead to bioconvection patterns that significantly impact flow dynamics, heat transfer, and mass transfer. The presence of microorganisms nit only enhances mixing but also stabilizes the fluid, making it relevant for applications in biotechnology and environmental engineering [6-7].

Considering heat generation alongside the presence of microorganisms adds another layer complexity. Heat generation within a system can arise from various sources, including chemical reaction, electrical currents, and the metabolic activities of microorganisms. This metabolic heat can alter the heat and mass transfer characteristics of the fluid. Some researcher shown that heat generation can increase the density of motile microorganisms and significantly impact temperature distributions in the fluid [8-9]. These findings are crucial for optimizing conditions in industrial and biological processes where temperature control is vital. Further investigations have demonstrated how heat generation affects fluid dynamics and the behavior of microorganisms [10-11]. The disturbance in fluid motion caused by changes in heat generation can generate heat that affects the microorganism boundary layer thickness. This influences the motile density of the microorganisms near the surface of the fluid. Heat generation also influences the fluid motion and the heat transfer within the system, which in turn affects the mass transfer processes. Additionally, examined the effects of heat generation on the stability and behavior of microorganisms in fluid flows, providing insights into applications such as cooling and heating processes, energy generation and thermal machines [12].

The impact of slip conditions on fluid dynamics and heat and mass transfer is another area of interest. Slip conditions occur when there is relative motion between a fluid and a solid boundary, influencing the transport phenomena in systems involving microorganisms. Some research has explored the effects of various slip conditions on bioconvection flow and the behavior of gyrotactic microorganisms [13-15]. These studies have shown that slip conditions can significantly affect heat and mass transfer rates, as well as the distribution and concentration of microorganisms. An increase in the velocity slip parameter influences heat transfer by initially enhancing the flow velocity, which can affect temperature distribution withing the boundary layer. This alteration in flow velocity can lead to changes mass transfer rates, with higher slip parameters initially boosting mass transfer but with diminishing effects as the parameter value increases. The impact of slip conditions on fluid flow, highlighting how these conditions influence the velocity profile, temperature distribution, and concentration boundary layer thickness[16-17]. These findings are critical for understanding and optimizing fluid-solid interactions in systems involving microorganisms.Recent research has continued to explore the interplay between slip conditions, heat generation, and the presence of microorganisms [18]. These studies underscore the importance of considering multiple factors simultaneously to accurately predict and optimize heat and mass transfer in complex systems.

In summary, the existing literature indicates that while significant progress has been made in understanding heat and mass transfer in the presence of microorganisms, there remains a gap in studies specifically addressing the combined effects of heat generation and slip conditions over a flat plate. This study aims to fill this gap by analyzing heat and mass transfer in boundary layer flow due to the swimming of motile gyrotactic microorganisms over a flat plate with slip conditions and heat generation. This research will provide valuable insights for optimizing industrial and biological processes where these factors play a crucial role.

Mathematical Formulation

Considers a boundary layer of viscous incompressible flow over a stretching flat surface along with the microorganisms. The temperature inside boundary layer is taken as $T, T_\infty$is the ambient temperature, T_w is wall temperature. U_∞ is free stream velocity and $u_w(x) = \varepsilon x$ is the plate velocity where ε is stretching parameter. C stands for mass concentration ,C_∞ denotes ambient mass concentration and C_W concentration at the surface. N is the density of microorganisms, N_∞ ambient microorganism density and N_W is the microorganism's density at the surface. The governing question for the boundary layer flow considered in the present study are written as [19-21].

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \tag{1}$$

$$v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x} = \upsilon\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\begin{aligned} v\frac{\partial T}{\partial y} + u\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \\ + \frac{Q_0}{\rho C_p}(T - T_\infty), \end{aligned} \tag{3}$$

$$v\frac{\partial C}{\partial y} + u\frac{\partial C}{\partial x} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$v\frac{\partial N}{\partial y} + u\frac{\partial N}{\partial y} + \frac{bW_c}{C_W - C_\infty}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right] = D_m\frac{\partial^2 N}{\partial y^2}. \tag{5}$$

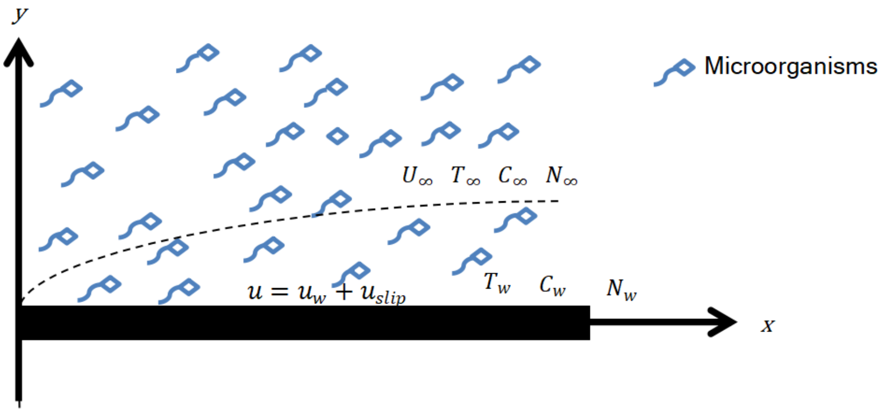

Figure 1 Schematic of a flow problem

Where *u* and *v* are the velocity components along *x* and *y* directions respectively. The kinematic viscosity is specified as υ and μ as dynamic viscosity. *k* is the thermal conductivity and ρ is the density of fluid, C_p is specific heat at constant pressure, D_B is brownian motion parameter, D_T is thermophoresis diffusion coefficient, τ is defined as heat capacity of the fluid.Q_0 is volumetric heat generation/absorption, bW_c is cell swimming speed and D_m represent diffusivity of microorganisms. $u_{slip} = A\frac{\partial u}{\partial y}$ where A is Navier slip coefficient. Navier slip coefficient quantifies the slip effect. It relates the slip velocity to the normal derivative of the fluid velocity at the solid boundary. The Navier slip coefficient is the parameter used to describe the slip condition in mathematical model of fluid near boundaries.

Along with the boundary conditions [19,22, and 23]

$$\begin{aligned} &u = u_w + u_{slip}, v = 0, T = T_w, C = C_w, N = N_w, at\ y = 0 \\ &u = U_\infty, T = T_\infty, C = C_\infty, N = N_{\infty,}\ at\ y \to \infty \end{aligned} \tag{6}$$

To reduce the dimensional partial differential equations (PDEs), (2)-(5) to nondimensional ordinary differential equations (ODEs), the following dimensionless variables are adopted [21 and 25]

$$\eta = \left(\frac{U_\infty}{2\upsilon x}\right)^{\frac{1}{2}} y, \psi = (2U_\infty \upsilon x)^{\frac{1}{2}} f(\eta)$$
$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_W - C_\infty}, \chi(\eta) = \frac{N - N_\infty}{N_W - N_\infty} \quad (7)$$

where θ is dimensionless temperature of fluid and ϕ is dimensionless concentration, and χ is dimensionless density of gyrotactic microorganisms, respectively. Also, stream function ψ is defined as $u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$.

Then transform a partial differential equation into the ordinary differential equation by employing a similarity transformation. This method utilizes a similarity variable, dimensionless parameters and the chain rule to systematically reduce the complexity of a PDEs to a more manageable ODEs form.

Using equation (7) and the stream function, the velocity components u and v are derived as

$$u = \frac{\partial\psi}{\partial y} = U_\infty f'(\eta), v = \frac{U_\infty y}{2x} f' - \left(\frac{U_\infty \upsilon}{2x}\right)^{\frac{1}{2}} f \quad (8)$$

Using equation (7), (8) equations (1)-(5) becomes

$$f''' + ff'' = 0, \quad (9)$$

$$\frac{1}{Pr}\theta'' + f\theta' + N_b\theta'\phi' + N_t\theta'^2 + E_c f''^2 + Q\theta = 0, \quad (10)$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Lef\phi' = 0, \quad (11)$$

$$\chi'' + Pr \cdot L_b f \chi' - Pe[\chi'\phi' + (\chi + \Phi)\phi''] = 0, \quad (12)$$

The similarity transformed dimensionless boundary conditions (6) are written as

$$f'(0) = \varepsilon + \gamma f''(0), f(0) = 0, \phi(0) = 1,$$
$$\theta(0) = 1, \chi(0) = 1, \quad (13)$$
$$f'(\eta) \to 1, \phi(\eta) \to 0, \theta(\eta) \to 0, \chi(\eta) \to 0 \; as \; \eta \to \infty$$

Where prime denotes differentiation with respect to η, $Pr = \frac{\upsilon}{\alpha}$ is prandtl number where $\alpha = \frac{k}{\rho c_p}$. $N_b = \frac{\tau D_B(C_W - C_\infty)}{\upsilon}$ is brownian motion parameter, $N_t = \frac{\tau D_T(T_W - T_\infty)}{T_\infty \upsilon}$ is thermophoresis parameter, $E_c = \frac{(U_\infty)^2}{C_p(T_W - T_\infty)}$ is eckert number, $Q = \frac{Q_0}{(\rho C)_f}\left(\frac{2x}{U_\infty}\right)$ is heat generation/absorption, $Le = \frac{\upsilon}{D_B}$ represents lewis number, $L_b = \frac{\alpha}{D_m}$ is bioconvection lewis number, $Pe = \frac{bW_c}{D_m}$ is bioconvection peclet number, $\Phi = \frac{N_\infty}{N_W - N_\infty}$ is similarity density gradient of gyrotactic microorganisms variable and γ is slip factor .

The skin friction (C_f), local nusselt number (Nu_x), and sherwood number (Sh_x) are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2}, Nu_x = \frac{xq_w}{(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (14)$$

Where,

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, q_m = -D_B\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{15}$$

Using similarity transformation given in (8), equations (14) together with (15) becomes

$$\begin{gathered} C_f(2Re_x)^{\frac{1}{2}} = f"(0), Nu_x\left(\frac{Re_x}{2}\right)^{\frac{1}{2}} = -\theta(0), \\ Sh_x\left(\frac{Re_x}{2}\right)^{\frac{1}{2}} = -\phi'(0) \end{gathered} \tag{16}$$

where $Re_x = \frac{U_\infty x}{v}$ is local reynolds number.

Results and discussion

The simplified governing equations (9) to (12) with boundary condition (13) are then numerically solved using the MATLAB solver which is called bvp4c. Bvp4c function in MATLAB utilizes a collocation method which is a type of finite difference method to solve boundary value problems(BVPs) for systems of ordinary differential equations. In the collocation method employed by bvp4c, the differential equations are evaluated at specific points within each subinterval of the domain, known as collocation points. By enforcing the satisfaction of the differential equations at these collocation points, the problem is transformed into a system of nonlinear algebraic equations that can be iteratively solved to obtain the solution to the BVP. This numerical approach is effective for handling BVPs with multiple boundary conditions and complex system of ODEs. Equation (9) - (12) are firstly written as a system of first order ordinary differential equations. Letting

$$\begin{gathered} \zeta_1 = f, \zeta_2 = f', \zeta_3 = f" \\ \zeta_4 = \theta, \zeta_5 = \theta', \\ \zeta_6 = \phi, \zeta_7 = \phi' \\ \zeta_8 = \chi, \zeta_9 = \chi' \end{gathered} \tag{17}$$

Substitutes equation (17) into equation (9) - (12), we have

$$\begin{gathered} \zeta_1' = f' = \zeta_2, \\ \zeta_2' = f'' = \zeta_3, \\ \zeta_3' = f"' = -\zeta_1\zeta_3, \\ \zeta_4' = \theta' = \zeta_5, \\ \zeta_5' = Pr[-\zeta_1\zeta_5 - N_b\zeta_5\zeta_7 - N_t\zeta_5^2 - E_c\zeta_3^2], \\ \zeta_6' = \phi' = \zeta_{7,} \\ \zeta_7' = -\frac{N_t}{N_b}\zeta_5 - Le\zeta_1\zeta_7, \\ \zeta_8' = \chi' = \zeta_{9,} \\ \zeta_9{}' = -Pr.L_b\zeta_1\zeta_9 + Pe[\zeta_9\zeta_7 + (\zeta_8 + \Phi)\zeta_7], \end{gathered} \tag{18}$$

The boundary conditions (13) becomes,

$$\begin{gathered} \zeta_2(0) = \varepsilon + \gamma\zeta_3(0), \zeta_1(0) = 0, \\ \zeta_6(0) = 1, \quad \zeta_4(0) = 1, \quad \zeta_8(0) = 1, \end{gathered} \tag{19}$$

$$\zeta_4(\infty) = 0, \qquad \zeta_2(\infty) = 1, \qquad \zeta_6(\infty) = 0, \zeta_8(\infty) = 0$$

Then, equation (18) together with the boundary condition (19) are solved using bvp4c.

To validate the computation that are computed by the bvp4c in the present study, the numerical a limiting case are computed and compared to the previous published results [21]. The results is demonstrated in Table 1 by using the choice of parameter values.

Table 1 Results validation for Nusselt number, Sherwood number and Skin friction with Pr=7, $N_b = N_t = E_c = 0.1$, $Q = 0$, $Le = 10$, $L_b = 0.1$, $Pe = 0.1$, $\Phi = 1, \gamma$=0.

ε	Nu_x		Sh_x		C_f	
	Fatima et al, (2023)	Present	Fatima et al, (2023)	Present	Fatima et al, (2023)	Present
0	0.3746	0.3749	1.1670	1.2895	0.4696	0.4698
0.1	0.4704	0.4707	1.3367	1.3362	0.4625	0.4627
0.5	0.7874	0.7882	1.8655	1.8637	0.3288	0.3289
1	1.0717	1.0695	2.3805	2.38012	0	0
2	1.2994	1.2982	3.3090	3.3180	-1.0191	-1.0184

The impacts of physical parameters including heat generation parameter Q slip parameter γ on the temperature profile $\theta(\eta)$, concentration $\phi(\eta)$, density of microorganisms $\chi(\eta)$, and velocity profile $f'(\eta)$ by fixing Pr=10, $N_b = 0.2$, $N_t = 0.1$, $E_c = 0.5$, $Le = 2$, $L_b = 0.8$, $Pe = 0.5$, $\Phi = 0.1$, and γ=0.1 are investigated.Figure 3 illustrates the effect of the variation of heat generation on the temperature profile. The heat generation denoted as Q plays a crucial role in determining the heat transfer characteristics of the system. When the heat generation Q parameter increases, it generally leads to an increase in the temperature within the boundary layer. This increase in the temperature reduces the viscosity of the fluid which can enhance the momentum diffusivity. As a result, the thermal boundary layer thickness typically increases. While in Figure 4 show the effects of heat generation Q on the concentration boundary layer thickness. When heat generation Q increase, it leads to a higher temperature within the boundary layer, which can enhance the rate of mass transfer. Hence, the concentration of particles tends to decrease due to the more vigorous mixing and diffusion by the increase thermal energy.

For Figure 5 and Figure 6, it show the effect of Q on the density of microorganisms and velocity respectively. In Figure 4, when the heat generation Q increases, it typically results in a reduction in the density of the microorganisms within the boundary layer. This is due to the enhanced temperature in the boundary layer, which affects the microorganism's environment. The higher temperature can cause the microorganisms to disperse more due to increased thermal energy, reducing their concentration near the surface. In Figure 6, as heat generation Q increase, the fluid's velocity tends to decrease. This is because the additional heat generation causes an increase in temperature, leading to a reduction in fluid density and viscosity. This results in a less effective momentum transfer within the boundary layer, thereby reducing the velocity.

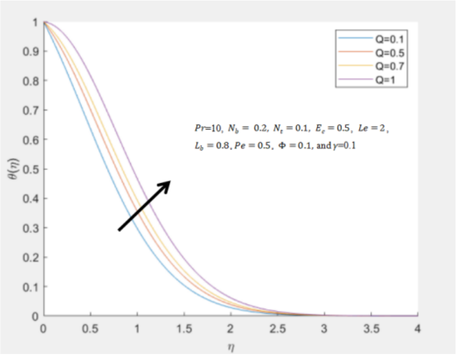

Figure 3 Effects of heat generation Q on $\theta(\eta)$

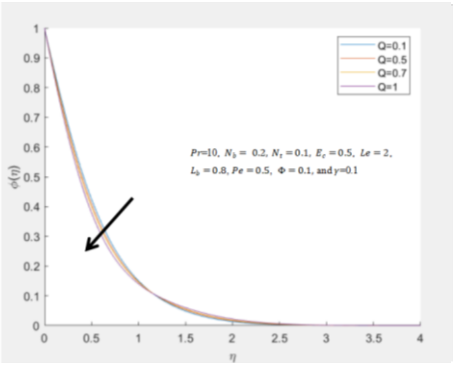

Figure 3 Effects of heat generation Q on $\phi(\eta)$

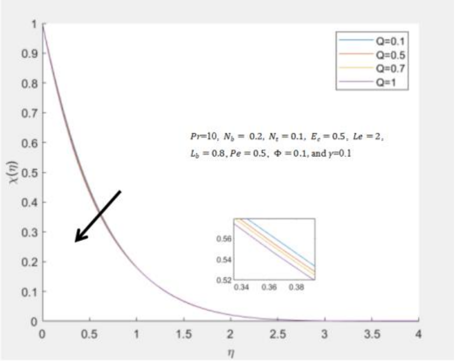

Figure 5 Effects of heat generation Q on $\chi(\eta)$

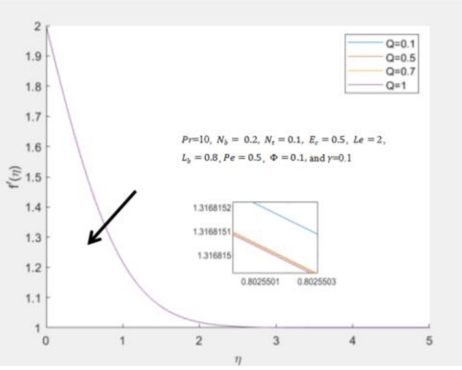

Figure 6 Effects of heat generation Q on $f'(\eta)$

The effect of slip parameter γ on the temperature profile $\theta(\eta)$, concentration $\phi(\eta)$, density of microorganisms $\chi(\eta)$, and velocity profile $f'(\eta)$ by fixing Pr=1, $N_b = N_t = E_c = 0.1$, $Le = 5$, $L_b = 1$, $Pe = 1$, $\Phi = 1$, and Q= -0.3 also discussed. Figure 7 show that as the variable slip parameter γ increase, the temperature within the boundary layer enhanced due to the increased difference in velocity between the stretching sheet and the neighboring fluid. While Figure 8, as the slip factor γ increases, it leads to a greater difference in velocity between the stretching sheet and the surrounding fluid. This results in a reduction in the velocity withing the boundary layer, impacting the mass transfer and concentration distribution. Consequently, the concentration boundary layer thickness decreases with an increase in the slip parameter γ. Figure 9 and Figure 10 illustrates the effect of slip parameter γ towards the density of microorganisms and velocity respectively. From Figure 9, an increase in slip parameter γ results the dimensionless density of microorganisms increases. This is because a higher slip parameter γ, reduces the velocity gradient near the stretching sheet which allowing more microorganisms to accumulate in this region. For Figure 10, as the slip parameter γ increases, the dimensionless velocity decreases. This reduction occurs because the increased slip allows for a greater difference in velocity between the stretching sheet and the neighboring fluid, leading to reduced velocity gradient near the surface.

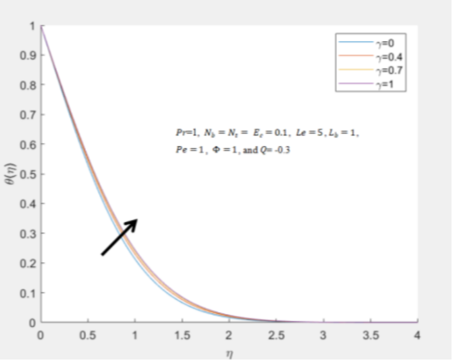

Figure 7 Effects of slip parameter γ on $\theta(\eta)$

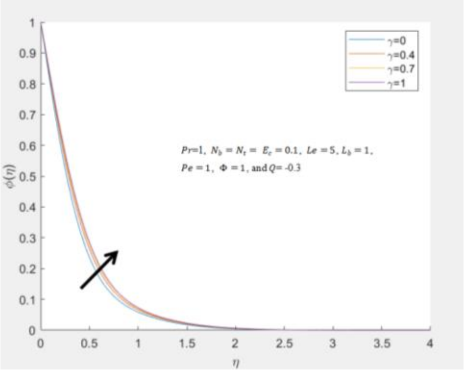

Figure 8 Effects of slip parameter γ on $\phi(\eta)$

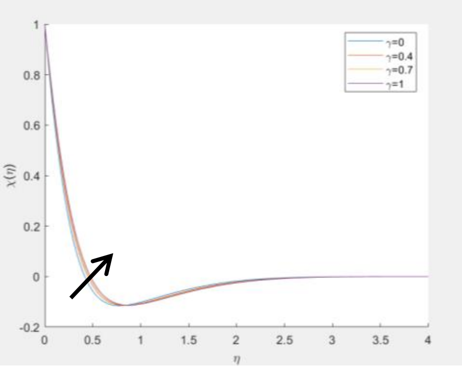

Figure 9 Effects of slip parameter γ on $\chi(\eta)$

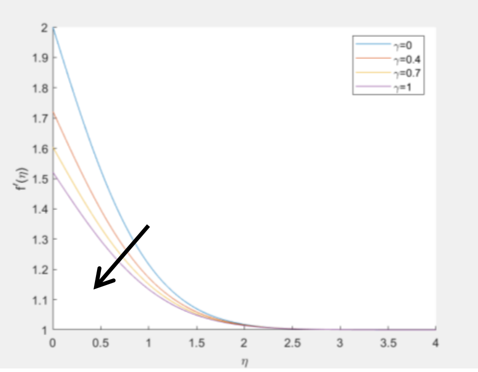

Figure 10 Effects of slip parameter γ on $f'(\eta)$

Table 2 and Table 3 Show the result of Skin friction coefficient, Sherwood number and Nusselt number for various of Q and γ by fixing other parameter. The nusselt number represents the ratio of convective to conductive heat transfer. When heat generation increases, the convective heat transfer becomes more dominant due to the higher thermal gradients. Thus, nusselt number increases with heat generation.While the sherwood number indicates the ratio of convective mass transfer to diffusive mass transfer. As heat generation increases, it indirectly affects the concentration gradients, enhancing the convective mass transfer. Consequently, sherwood number increases with higher heat generation. While with increasing slip, the relative motion at the boundary reduces the effective velocity gradients at the surface. This diminishes the convective heat and mass transfer rates, leading to a decrease in both nusselt and sherwood numbers. The skin friction coefficient is directly related to the shear stress at the wall. With a higher slip parameter, the wall shear stress increases due to the enhanced velocity gradient, resulting in a higher skin friction coefficient.

Table 2 Skin friction coefficient, Sherwood number and Nusselt number for various of Q by fixing Pr=1, $N_b = 0.2$, $N_t = 0.1$, $E_c = 0.5$, $Le = 2$, $L_b = 0.8$, $Pe = 0.5$, $\Phi = 0.1$, and γ=0.1

Q	C_f	Sh_x	Nu_x
0.1	**-0.9246**	**1.6164**	**0.3427**
0.5	**-0.9246**	**1.9779**	**0.4663**
0.7	**-0.9246**	**2.1941**	**0.9538**
1.0	**-0.9246**	**2.5883**	**1.8493**

Table 3 Skin friction coefficient, Sherwood number and Nusselt number for various of γ Pr =1, $N_b = N_t = E_c = 0.1$, $Le = 5$, $L_b = 1$, $Pe = 1$, $\Phi = 1$, and Q= -0.3

γ	C_f	Sh_x	Nu_x
0	**-1.0359**	**2.0974**	**0.9943**
0.4	**-0.7055**	**1.9396**	**0.9756**
0.7	**-0.5740**	**1.8717**	**0.9643**
1.0	**-0.4854**	**1.8238**	**0.9554**

Conclusion

In this research, heat and mass transmission in a boundary layer flow due to swimming of motile gyrotactic microorganisms over a flat plate with heat generation and slip velocity has been studied. The governing equations consist of continuity, momentum, energy, concentration and microorganisms. The governing partial differential equations are transformed into a set of ordinary differential equations by using similarity transformation. Then, the transformed governing equations and boundary conditions are then solved using bvp4c in MATLAB software. To ensure the accuracy of the algorithm developed in MATLAB software, the numerical results of Skin friction coefficient and

Nusselt number as well as Sherwood number are studied. The significant outcomes of the study as follow:

- Temperature profiles increase with the increment heat generation parameter.
- The increment in heat generation indicates a reduction in the density of microorganisms within the boundary layer, while the fluid’s velocity tends to decrease.
- The thermal boundary layer thickness increase as the slip velocity increase.
- An increase in slip factor results the dimensionless density of microorganisms increasing, while the dimensionless velocity decreases.

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