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Dynamical Analysis of a Predator-Prey Model with Holling Type II Functional Response and Harvesting

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Abstract

In this study, we examine the dynamical analysis of a predator-prey model that incorporates a Holling type II functional response, prey refuge, and harvesting in both populations. The objective of this study is to analyze the predator-prey interactions and determine the impact of overharvesting on the ecosystem. The stability analysis of each equilibrium is performed in this research. We also conduct a bifurcation analysis. Numerical simulations are performed to illustrate the dynamical behavior of the modified predator-prey model. Using MATLAB packages, we generate a bifurcation diagram with respect to the harvesting parameter to explore how variations in this parameter affect the model's dynamics. Additionally, time series are generated to validate the results obtained from the bifurcation analysis.

Keywords Prey; Predator; Harvesting; Holling Type II

Introduction

A predator-prey model is a model in which two distinct species are included and their interactions are shown. This study covers the predator-prey system with Holling type II functional response, as well as providing a carrying capacity for prey and harvesting in both populations. The predator-prey model can be analyzed dynamically to predict potential future scenarios. In the limit of huge population sizes, the most straightforward and widely used way to describe predator-prey dynamics is by ordinary differential equations (ODEs) of the Lotka–Volterra type with continuous population density variables. Nonlinear 'functional responses' of this type were originally proposed by Holling based on a general argument concerning the allocation of a predator's time between two activities: 'prey searching' and 'prey handling' [1]. The modified predator-prey model, which includes the Lotka-Volterra framework and incorporates non-linear system equations with logistic growth for both species, and the carrying capacities of prey and predator, has been discussed by [2].

There are numerous biological factors, such as predation, migration, and refuge, that significantly alter the dynamic behaviour of ecological systems. These factors interact in complex ways to influence population dynamics, stability, and the overall health of ecosystems. The effects of harvesting have played a significant role in prey-predator models. Studying the dynamics of interacting population with harvesting is becoming more and more important, which is closely related to management of renewable resources [3]. Harvesting plays a critical role in controlling the size of prey or predator populations and averting the extinction of a species through predation. Also, harvesting comes in several forms, such as continuous, nonlinear, and constant harvesting. According to Mortuja et al (2021), prey-predator systems with nonlinear prey harvesting and square root functional response can coexist and maintain ecological balance if the harvesting rate is chosen at a proper value below the maximum sustainable yield [4]. In general, if a species is harvested frequently and regularly, a constant rate harvesting strategy can be adopted to maintain population stability. However, due to seasonal variations and economic considerations, periodic harvesting emerges as an effective strategy for managing species that are harvested infrequently, allowing for population recovery and adaptation to changing conditions.

Furthermore, some researchers have developed the Lotka-Volterra model with various assumptions. For instance, [5] assumed interactions between prey and predators using Holling type I functional response, while [6] and [7] employed Holling type II, and [8] utilized Holling type III. These studies emphasize that these populations play a vital role in human life. For Holling type II, the case is the gradient of the curve decreases monotonically with increasing prey density and saturating at a constant value of prey consumption.

The main objective of this research paper is to study the dynamical analysis of prey-predator model with Holling type II and Harvesting. The prey species and the predator species are subject to a certain rate of harvesting. We want to study the effect of harvesting on both prey and predator populations. We aim to study the effect of harvesting on both prey and predator populations, examining how different harvesting strategies influence their dynamics. By analyzing the impact of constant and periodic harvesting, we seek to understand the long-term implications for ecosystem stability and health.

Model Formulation

In this work, we consider a predator prey model with functional responses Holling type III and harvesting derived by [9]. If the functional responses become Holling II and provides refuge in the prey. Then the model is as follows:

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \frac{(1-n)mxy}{1+x} - a_1Q_1x,$$
$$\frac{dy}{dt} = \frac{(1-n)cxy}{1+x} - by - a_2Q_2y, \tag{1}$$

Where $x(t), y(t)$ represent the number of prey and predator population at time t, respectively. Here r and c are growth rate of prey and predator, a_1 and a_2 are coefficient of prey and predator, Q_1 and Q_2 are harvest rate of prey and predator, K is carrying capacity, n is prey refuge rate, m is predation rate and b is the natural death rate of the population.

Non-Dimensionalization

To reduce the parameter and simplify the model, non-dimensionalization is carried out. First of all, we let the variable as follow:

$$X = \frac{x}{x_0}, Y = \frac{y}{y_0}, T = \frac{t}{t_0}$$

The dimensional system (1) is transformed into:

$$\frac{dX}{dT} = \alpha X(1-X) - \frac{XY}{1+kX} - \beta X,$$
$$\frac{dY}{dT} = \frac{vXY}{1+kX} - Y - \mu Y, \tag{2}$$

Where

$$\alpha = \frac{r}{b}, \beta = \frac{a_1Q_1}{b}, v = \frac{(1-n)ck}{b}, \mu = \frac{a_2Q_2}{b}$$

In the system (2):

α: ratio of growth rate of prey over natural death rate of the population
β: ratio of product coefficient of prey and harvest rate of prey over natural death rate of the population
v: ration of product prey refuge rate, growth rate of predator and carrying capacity over natural death rate of the population
μ: ratio of product coefficient of predator and harvest rate of predator over natural death rate of the population

Steady state, Equilibria, and Stability Analysis

To obtain equilibrium point for system (2), we can obtain the solution by letting $\frac{dX}{dT} = 0, \frac{dY}{dT} = 0$. From this model, there are three equilibria that can be obtained.

From model (2) we have equilibrium points as follow:

i. The extinction of all population's equilibrium points: $E_1 = (0,0)$.
ii. The predator free equilibrium points: $E_2 = \left(1 - \frac{\beta}{\alpha}, 0\right)$.
iii. The coexistence equilibrium points: $E_3 = \left(-\frac{\mu+1}{k+k\mu-v}, \frac{v(\alpha k\mu-\beta k\mu+\alpha k+\alpha\mu-\alpha v-k\beta+\beta v+\alpha)}{(k+k\mu-v)^2}\right)$.

Next, the stability of equilibrium point can be obtained by using Jacobian Matrix, where the system can be generalized as follows:

$$J(E^*) = \begin{bmatrix} \alpha(1-2X) + \frac{XYk}{(1+kX)^2} - \frac{Y}{1+kX} - \beta & -\frac{X}{1+kX} \\ \frac{vY}{1+kX} - \frac{vXYk}{(1+kX)^2} & \frac{vX}{1+kX} - (1+\mu) \end{bmatrix}$$

From the extinction of all population equilibrium point: $E_1 = (0,0)$, we can obtain following Jacobian Matrix $J(E_1) = \begin{bmatrix} \alpha-\beta & 0 \\ 0 & -1-\mu \end{bmatrix}$ by substitute the value of E_1 .From the $J(E_1)$, we can obtain characteristic equation as follow:

$$\lambda^2 + (\beta - \alpha + 1 + \mu)\lambda - (\alpha - \beta)(1 + \mu) \tag{3}$$

where λ is the eigenvalue. We obtain the eigenvalue, $\lambda_1 = \alpha - \beta, \lambda_2 = -(1+\mu)$. Based on stability of the eigenvalues from Jacobian matrix, E_1 is stable if $\lambda_1 < 0$ and $\lambda_2 < 0$. Therefore, in order to let E_1 to remain unstable, $\lambda_1 = \alpha - \beta > 0$.

From the predator free equilibrium point: $E_2 = \left(1 - \frac{\beta}{\alpha}, 0\right)$, we can obtain following Jacobian Matrix $J(E_2) = \begin{bmatrix} -\alpha+\beta & -\frac{\alpha-\beta}{\alpha+k(\alpha-\beta)} \\ 0 & \frac{v(\alpha-\beta)}{\alpha+k(\alpha-\beta)} - (1+\mu) \end{bmatrix}$ by substitute the value of E_2. From the $J(E_2)$, we can obtain characteristic equation as follows:

$$\lambda^2 + \frac{1}{\alpha K - K\beta + \alpha}\big((\alpha-\beta)(\alpha K - K\beta + \alpha) + \alpha K\mu - \beta K\mu + \alpha K + \alpha\mu - \alpha v - \beta K + \beta v + \alpha\big)\lambda + \frac{1}{\alpha K - K\beta + \alpha}((\alpha-\beta)(\alpha K\mu - \beta K\mu + \alpha K + \alpha\mu - \alpha v - \beta K + \beta v + \alpha)) \tag{4}$$

We obtain the eigenvalue, $\lambda_1 = -\alpha + \beta,\ \lambda_2 = -\frac{(\alpha k\mu - k\beta\mu + \alpha k + \alpha\mu - \alpha v + k\beta + \beta v + \alpha)}{\alpha + k(\alpha-\beta)}$. For the stability of E_2, those condition need to fulfil:

i. $\beta - \alpha < 0$
ii. $(\alpha k\mu - k\beta\mu + \alpha k + \alpha\mu - \alpha v + k\beta + \beta v + \alpha) > 0$

If condition ii holds, the equilibrium of E_2 is guaranteed be stable node, or else the equilibrium of E_2 are guaranteed to be unstable node if $(\alpha k\mu - k\beta\mu + \alpha k + \alpha\mu - \alpha v + k\beta + \beta v + \alpha) < 0$.

The equilibrium E_3 represents the coexistence of both prey and predator species. To ensure the feasibility and positivity of the steady state, the following conditions should be fulfilled:

i. $\alpha > \beta$
ii. $k\mu + k - v < 0$

It is complicated to investigate the stability of E_3 by using Jacobian Matrix. Therefore, Routh Hurwitz criterion is being considered to examine the stability of coexistence equilibrium point. In accordance with Routh Hurwitz criterion, the equilibrium points of E_3 is asymptotically stable if $a_0 > 0, a_1 > 0, a_2 > 0$. The characteristic equation of $J(E_3)$ is given by:

$$a_0\lambda^2 + a_1\lambda + a_2 = 0. \tag{5}$$

where

$a_0 = 1$

$a_1 = \frac{\alpha-\beta(k^2\mu^2+2k^2\mu-k\mu v+k^2-kv)+\alpha(k\mu^2+2k\mu+k+v+\mu v)}{v(v-K-K\mu)}$

$a_2 = \frac{\alpha-\beta(k^2\mu^3+3k^2\mu^2-2k\mu^2v+3k^2\mu-4k\mu v+\mu v^2+k^2-2kv+v^2)+\alpha(k\mu^3+3k\mu^2-k-v-2\mu v)}{v(v-k-k\mu)}$

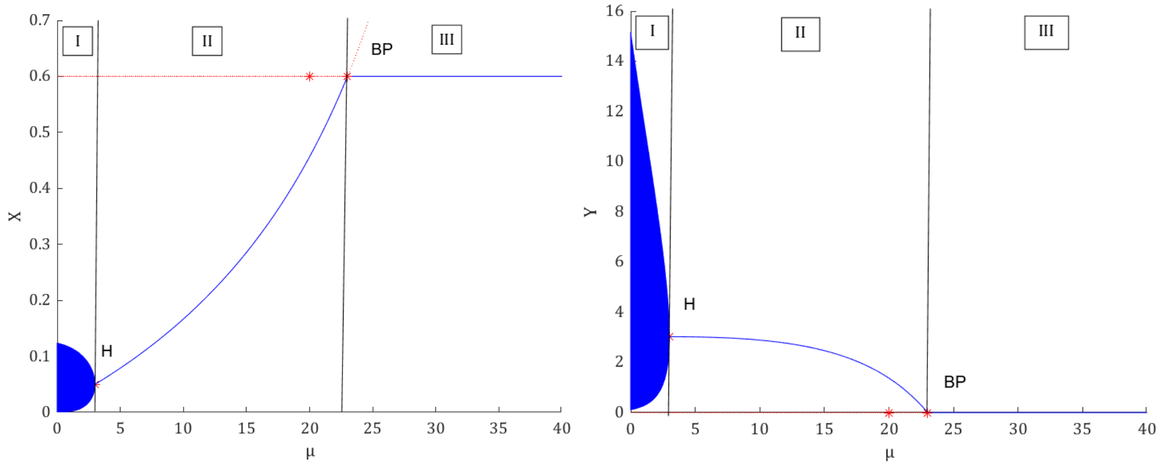
To ensure the stability of coexistence equilibrium point, those necessary conditions need to be fulfilled:

i. $\mu < \frac{(\alpha-\beta)(v-k\mu-k)}{\alpha}$

ii. $\mu < \frac{v((\alpha-\beta)k-\alpha)}{k((\alpha-\beta)k+\alpha)} - 1$

Bifurcation Result and Analysis

To examine the dynamical behaviour of system (2), numerical simulations are performed by using MATCONT package which is the extension package of MATLAB. In order to get result of bifurcation diagram, we set the parameters $\alpha = 5, \beta = 2, \mu = 3.01, v = 88, k = 2$. Moreover, we set μ and k as free parameter in the system (2) to examine the effect of harvesting on predator and carrying capacity. Figure 1 (a) and (b) shows the bifurcation diagram with respect to harvesting on predator parameter, μ and figure 2 (a) and (b) shows the bifurcation diagram with respect to carrying capacity, k. For illustrative purposes, the blue solid lines represent stable steady states, while the red dashed line indicates unstable steady states.



(a) (b)

Figure 1 Bifurcation curve with respect to predation harvesting parameters μ with fixed parameters $\alpha = 5, \beta = 2$ $v = 88, k = 2$ for (a) prey, x and for (b) predator, y respectively.

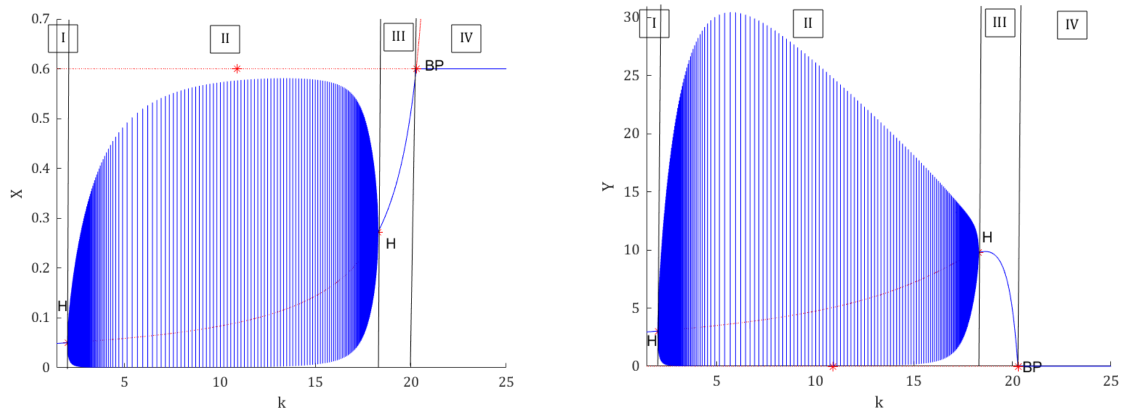
By referring to figure 1 (a) and (b), there is Hopf bifurcation at $\mu = 3.0000$ where both prey and predator populations are oscillating when $\mu < 3.0000$. Also, we can examine a transcritical bifurcation at $\mu = 23.0000$, where the stability of two steady states is interchanged. This because when the value of μ greater than 3.0000, the stability of E_3 are gained, and when the value of μ greater than 23.0000, the steady-state E_3 loses stability while E_2 gains stability, leading to an interchange of the two steady-state branches.

From both diagrams, it's clear that the Hopf and transcritical bifurcation points divide the positive quadrant into three regions: I, II, and III. Region I corresponds to low predator harvesting, region II to moderate predator harvesting, and region III to high predator harvesting. Each region contains only one stable equilibrium point.

From figure 1 (a) and (b), the population of both prey and predator species are oscillating at low level of harvesting on the predator μ. When predator harvesting is low, the prey population experiences significant growth, subsequently providing abundant food resources for the predator population to thrive. As the prey population becomes overly saturated, it reaches a point where it can no longer sustain its numbers, causing a decline. This decrease in prey availability leads to a subsequent decline in the predator population due to the reduced food supply. Following this reduction, the prey population gradually begins to recover, initiating a cyclic pattern of growth and decline. This sequence of oscillations continues over time, with both prey and predator populations fluctuating until they eventually stabilize at a constant amplitude, reaching a dynamic equilibrium.

When harvesting activities are carried out at intermediate level which is the value of μ fall in between 3.0000 and 23.0000, this means that the number of predators harvested increases, and gives more space for prey to growth. After that, the population of predator are increases after the prey population increases as there providing abundant food resources for the predator to growth. Next, the prey population will decrease as they are consumed by the predators. At this stage, both prey and predator species can coexist in the environment.

When harvesting activities are carried out at high levels which is the value of μ is greater than 23.0000, the predator free equilibrium of the system (2) will converge, which show that the equilibrium point E_2 is asymptotically stable while at same time, the equilibrium points of E_3 will become unstable. This is because the predator is overly harvested, causing it to become extinct from the environment and the prey population continue to growth without any predation. The prey population will stop growing when it reaches high saturation.



(a) (b)

Figure 2 Bifurcation curve with respect to carrying capacity parameters k with fixed parameters $\alpha = 5, \beta = 2\ \ v = 88, \mu = 3.01$ for (a) prey, x and for (b) predator, y respectively.

By referring to figure 2 (a) and (b), there is Hopf bifurcation with two Hopf point at $k = 2.0011$ and $k = 18.2773$. This will cause both prey and predator population to start oscillating at $k = 2.0011$ and slowly stop to oscillate at $k = 18.2773$. Plus, we can observe a transcritical bifurcation at $k = 20.2785$ where the stability of two steady states is interchanged. When the value of k falls in region III, the equilibrium points of E_3 will gain stability and when the value of k falls in region IV, the transition occurs as the steady-state E_3 loses stability while E_2 gains stability, leading to an interchange of the two steady-state branches.

At low level of carrying capacity, both prey and predator populations will coexist. At a low level of carrying capacity, both prey and predator populations can coexist. This means that the environment can support a limited number of individuals from both species without depleting resources. In such a scenario, the prey population is kept in check by the predators, preventing it from exceeding the carrying capacity of the environment. Similarly, the predator population is regulated by the availability of prey, ensuring that it does not overconsume the prey population to the point of extinction. This balanced interaction allows for a stable coexistence where both species maintain sustainable population levels.

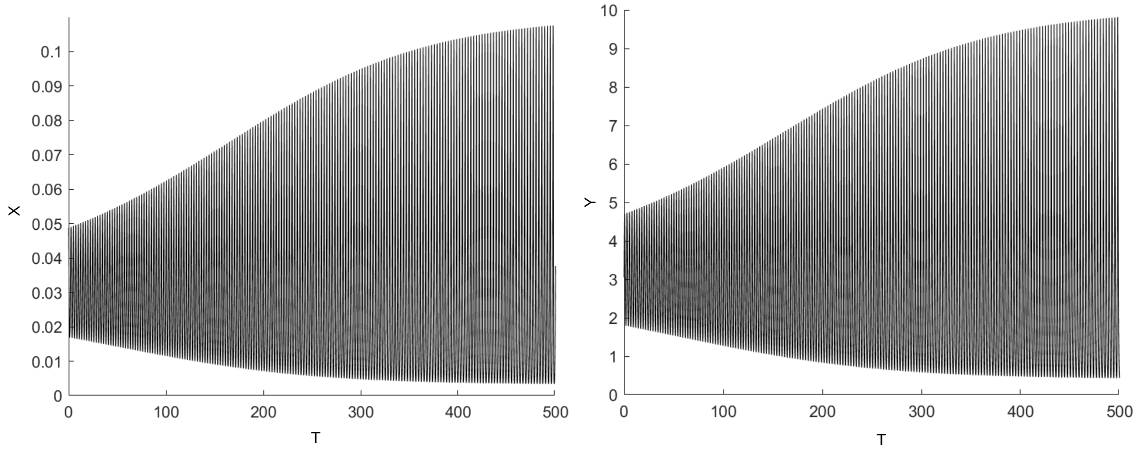
At intermediate level of carrying capacity, the environment can support a moderate number of prey individuals. In this scenario, the prey population has enough resources to grow and increase in number. As the prey population expands, it provides ample food resources for the predator population, allowing the predators to growth. As the prey population becomes overly saturated, it decreases, leading to a decline in the predator population. Following this, the prey population gradually recuperates, initiating a cyclic pattern. As it approaches a Hopf point $k = 18.2773$, both prey and predator populations are slowly to stop oscillating.

At high level of carrying capacity, the environment can support a large number of preys. In this scenario, the prey population reaches a point where it can grow significantly, providing a substantial and consistent food source for the predator population. As a result, the predator population also increases. Eventually, both populations reach a stable coexistence.

At very high level of carrying capacity, the value k falls in region IV, the predator free equilibrium of the system (2) will converge, which show that the equilibrium point E_2 is asymptotically stable while at same time, the equilibrium points of E_3 will become unstable. This is because of the exponential increase in prey populations. When prey populations growth due to abundant resources, their numbers surge exponentially. Initially, this surge in prey abundance can support a rapid expansion in the predator population, as there is ample food supply available. However, as prey populations continue to flourish, the natural habitats of predators become increasingly crowded and there is a loss of space for growth, causing it to become extinct from the environment and the prey population continue to growth without any predation. The prey population will stop growing when it reaches high saturation.

Result of Time Series

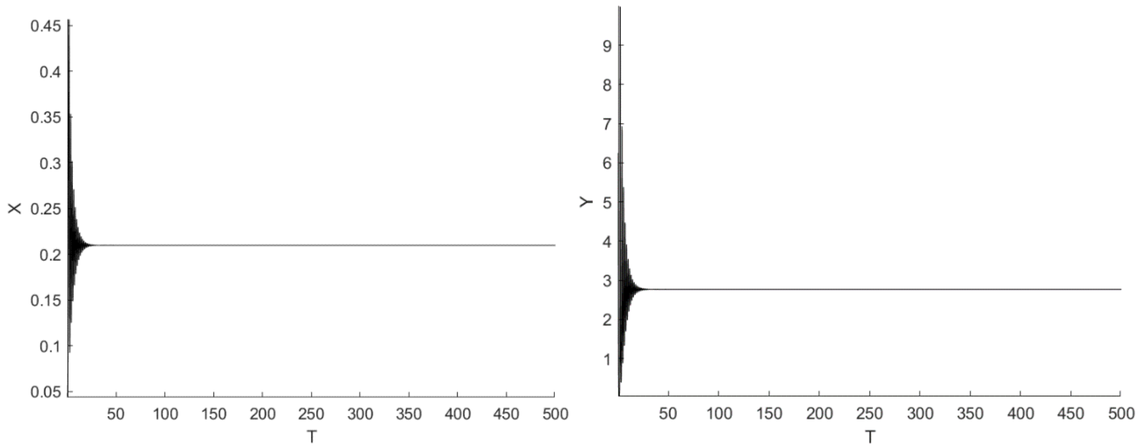
To explore the dynamic changes in prey and predator populations over time, time series plots were generated using the MATCONT package. These plots utilize the parameter values outlined based on Figure 1 and 2, with the μ values set to 1.5, 12, and 35, and k values set to 1.6, 10, 19 and 21.



(a) (b)

Figure 3 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ k = 2$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $\mu = 1.5$ for (a) prey and (b) predator

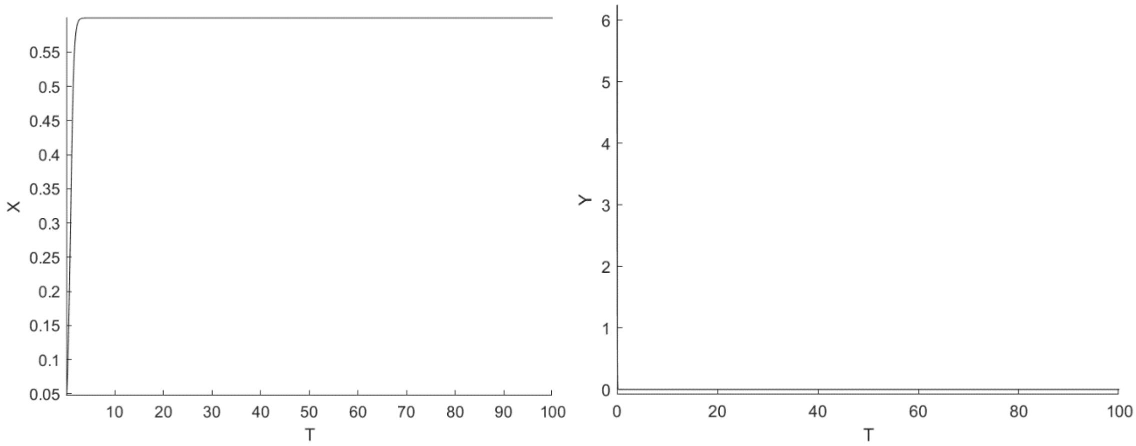
In figure 3 (a) and (b), it is describing the behaviour of both prey and predator species when the predator harvesting activities μ are carried out at low level. As we observed from the figures, both prey and predator populations are oscillating. Initially the prey population decreases due to predation leading to predator populations to growth. Moreover, harvesting activities reduce the predator population allowing the prey to growth. However, the harvesting on predator is minimal which allow the predator to thrive due to high population of prey. Consequently, the prey population declines once more. This cycle continues until reaching a maximum population constrained by environmental factors.



(a) (b)

Figure 4 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ k = 2$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $\mu = 12$ for (a) prey and (b) predator

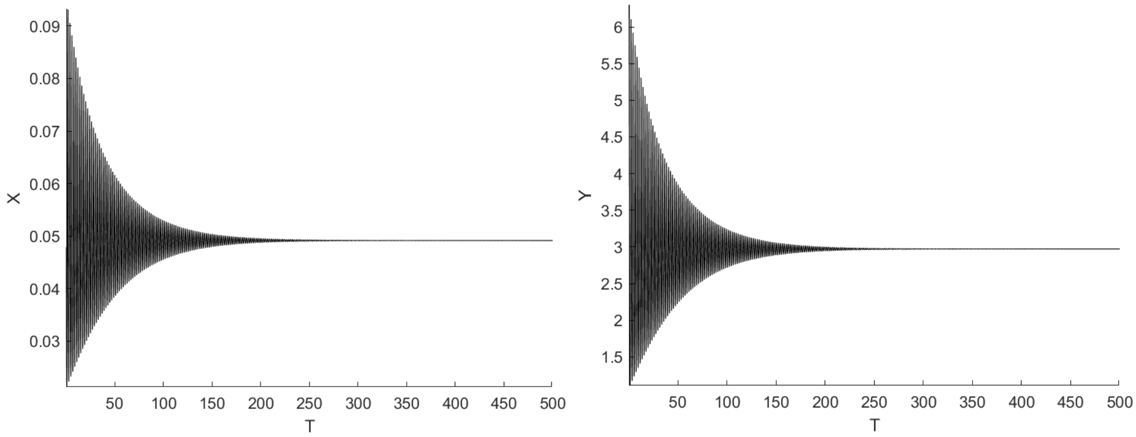
Figures 4 (a) and (b) describes the phenomena and behaviours of both prey and predator species at an intermediate level of predator harvesting activities, where the value of μ fall between 2.999994 and 23. When μ = 15, both prey and predator species coexist. Initially, the prey population decreases as it is consumed by the predator. After that, predator populations reduced due to harvesting activities, leading to increase the population of prey because of predation rate is minimal. This alternating pattern between the prey and predator populations continues until reaching a stable level.



(a) (b)

Figure 5 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ k = 2$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $\mu = 35$ for (a) prey and (b) predator

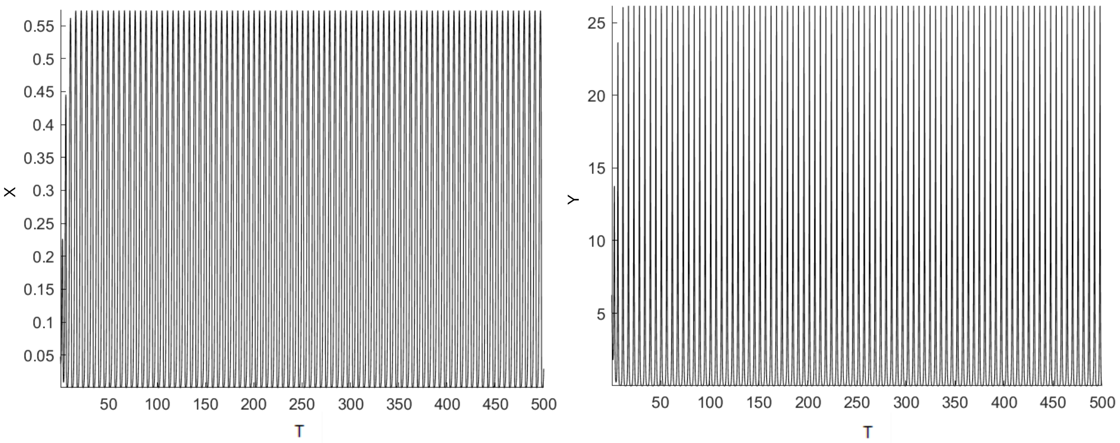
When the harvesting parameter μ surpasses 23, the predator population confronts extinction, as depicted in Figure 5 (a) and (b). This is attributed to excessively high predator harvesting, resulting in a decline in the predator population, while the prey population continues to thrive. However, despite the prey population's growth, it becomes constrained by the environmental carrying capacity.



(a) (b)

Figure 6 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ \mu = 3.01$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $k = 1.6$ for (a) prey and (b) predator

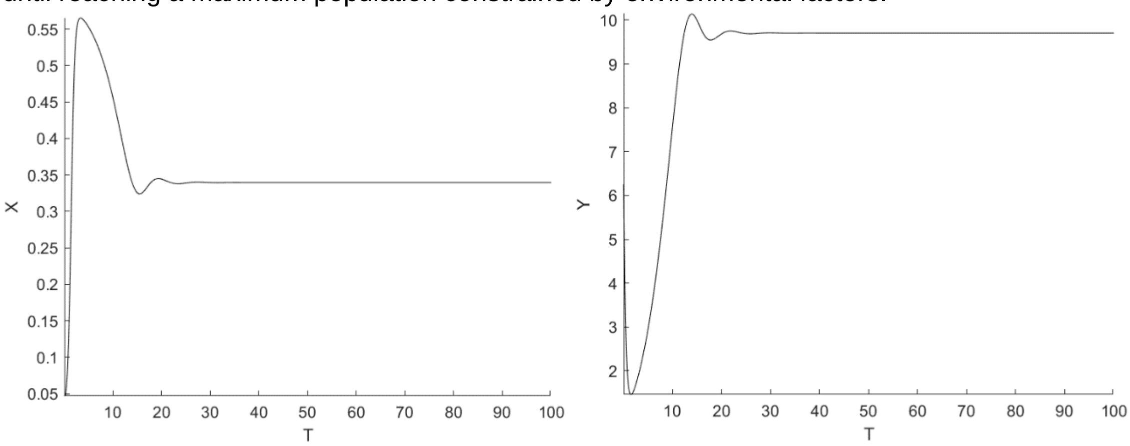
Figures 6 (a) and (b) illustrate the phenomena and behaviours of both prey and predator species at a low level of prey carrying capacity, where the value of k is less than 2.0011. At $k = 1.6$, both prey and predator species exhibit coexistence. The carrying capacity at this level does not influence the existence or stability of the coexistence equilibrium point.



(a) (b)

Figure 7 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ \mu = 3.01$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $k = 10$ for (a) prey and (b) predator

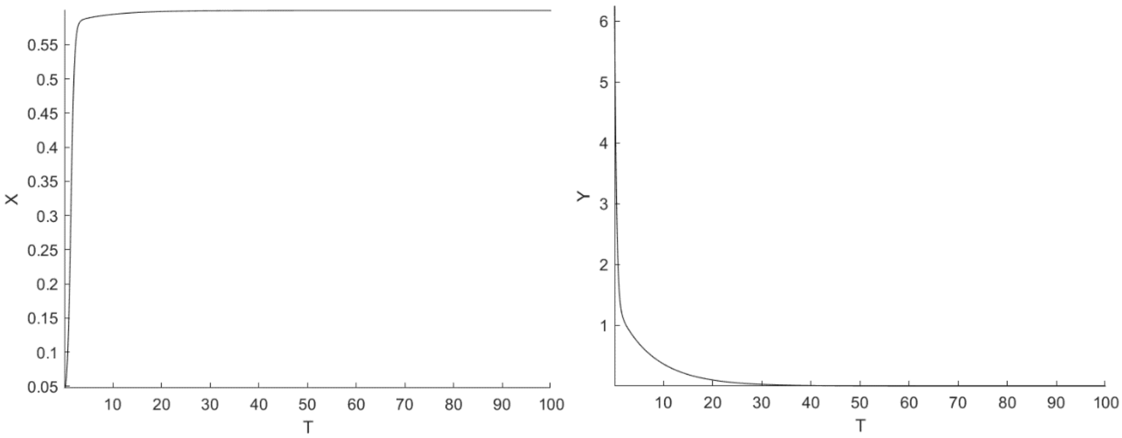
In Figure 7 (a) and (b), it's apparent that the carrying capacity of prey activities becomes uncontrollable when the carrying capacity of prey parameter k, falls between 2.0011 and 18.2773. This leads to oscillations in both prey and predator populations. At carrying capacity k is equal 10, the prey population has enough resources to grow and increase in number. As the prey population expands, it provides ample food resources for the predator population, allowing the predators to growth. As the prey population becomes overly saturated, it decreases, leading to a decline in the predator population. Following this, the prey population gradually recuperates, initiating a cyclic pattern. This cycle continues until reaching a maximum population constrained by environmental factors.



(a) (b)

Figure 8 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ \mu = 3.01$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $k = 19$ for (a) prey and (b) predator

By referring to figure 8 (a) and (b) describes the phenomena and behaviours of both prey and predator species at a high level of carrying capacity of prey, where the value of k less than 18.277346. When k = 19, both prey and predator species coexist. The carrying capacity at this level does not affect the existence or stability of the coexistence equilibrium point.



(a) (b)

Figure 9 Time series plot of model with parameter $\alpha = 5,\ \beta = 2,\ v = 88,\ \mu = 3.01$ and the initial condition $(x_0, y_0) = (0.04827, 6.30294)$ at $k = 19$ for (a) prey and (b) predator

From figure 9 (a) and (b), when k = 21, both populations of prey and predator are converging to equilibrium point, E_2. When the level of carrying capacity is too high, the population of prey increases, and the population predator increases but over a short period of time. Following this, the predator population begins to decrease and eventually becomes extinct because of the prey population reaching excessive levels. The surge in the prey population leads to intensified competition for limited resources and significant habitat loss, which in turn creates an unsustainable environment for predators. This increased competition among prey species for food and living space not only depletes the resources available for the predators but also contributes to an overall decline in the quality of the habitat. Therefore, the population of prey species will increase because of no predation. The prey population will stop growing when it reaches high saturation.

Conclusion

This study analyzed the prey-predator population model with Holling type II functional response and harvesting for both prey and predator populations. The stability properties of the proposed prey-predator model are evaluated at predator free equilibrium by checking the eigenvalues of the characteristic polynomial and coexistence equilibrium using the Routh-Hurwitz criterion.

The effects of harvesting and carrying capacity on prey-predator model are observed with different values of the harvesting parameter, μ and carrying capacity parameter, k.

Moreover, the proposed prey-predator model is numerically simulated using the ODE45 function in MATLAB, generating bifurcation diagram and the time series plot that helps us to observe the dynamical behaviours of prey and predator population.

According to the steady state diagrams, there is a phenomenon called Hopf bifurcation at low level of harvesting activity that causes the oscillation in prey and predator populations. The phenomenon is not suggested as it can cause an extinction to either one or both population if there exists an external factor that occur to the environment. At intermediate level of harvesting, both prey and predator populations are able to coexist. The high levels of harvesting should be avoided as it can lead the population of predator to extinction.

Referring to the steady state diagrams for carrying capacity on prey, both species may coexist at low levels of carrying capacity on prey. At intermediate level of carrying capacity on prey, there is phenomenon of Hopf bifurcation. However, there are two Hopf point which means there is closed Hopf. At first Hopf point, the prey and predator are oscillating and slowly to stop oscillates as it is approaching the second Hopf point. At high level of carrying capacity on prey, both prey and predator may coexist, but the population of prey is increasing, while the population predator is decreasing and until it is driven to extinct.

Based on numerical simulation analysis, it has been successfully demonstrated that the dynamical behaviours of this prey-predator system are primarily dependant on some critical parameters. It is important to highlight how the control parameter μ and k affect the complexity of the system (2), which can lead this prey-predator system to emerge in Hopf and transcritical bifurcations. These findings also demonstrate the direct or indirect influence of various control parameters on the dynamic complexity of the prey-predator system.

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