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Analytical Solution of Blood and Body Temperature Dynamics Using One Dimensional Pennes Bioheat Equation

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Abstract
This study focuses on the development of a theoretical model to simulate thermal interactions between blood and tissue in the human body during blood cooling and rewarming processes. The problem addressed is needed for accurate prediction of body and blood temperature changes during clinical applications such as managing systemic hypothermia or hyperthermia in patients with brain injuries or during cardiac surgeries. The methodology integrates the Pennes bioheat equation with a heat transfer equation for blood, creating a model that accounts for the energy balance among blood and tissue, including the influence of external cooling or rewarming. The research process involved validating the model's accuracy through theoretical analysis and comparison with a simple lumped analysis. The results show that the model accurately represents how heat is transferred between blood and tissue, making it reliable for predicting temperature changes in medical situations. However, limitations include the simplified body geometry and assumptions of constant physiological parameters, which may not fully capture the complex dynamics of heat transfer in a real human body. Additionally, the model does not consider the rewarming effects of venous blood due to counter current heat exchange, and the explicit numerical method imposes restrictions on the time step to avoid oscillations. Overall, this research offers a strong basis for developing better temperature management strategies in medical treatments and physiological studies.
Keywords: Pennes bioheat equation; heat transfer; body temperature prediction; numerical modelling.

1. Introduction

Bioheat transfer is an important area that combines engineering, biology, and medicine to help us understand how heat moves inside living things. Seminal works like the Pennes bioheat [7], Weinbaum and Jiji's simplified bioheat equation [14] and studies on heat transfer in perfused biological tissues [4] provide foundational frameworks for analysing heat distribution in biological systems. Analytical solutions, exemplified in [1] aid in predicting tissue temperature profiles, while numerical simulations, as seen in [8] enhance comprehension of thermal dynamics at the tissue level.

This project investigates the effects of sinusoidal heat flux on transient temperature distribution in biological tissue, addressing challenges in analytical solutions for the Pennes bioheat equation under periodic heat flow conditions. By deriving exact solutions, this study aims to deepen understanding of tissue response to periodic heat flux and its practical implications in medical physics and engineering. Through this research, insights into thermal interactions between blood and tissue, numerical methods for solving the Pennes bioheat model analytically, and accurate prediction of temperature changes during thermal interventions will be garnered, contributing to advancements in medical engineering and physics and ultimately improving patient care and treatment outcomes.

Bioheat transfer combines engineering, biology, and medicine to study how heat moves through biological systems. This understanding is crucial for treatments like hyperthermia for cancer and thermal therapy. Early studies, like Bernard's tests in 1876 [13], highlight the long-standing interest in the

relationship between blood vessels and tissue. Advances in computational techniques have enabled the development of complex models to analyse temperature distributions in biological systems.

Theoretical models, like continuum and one-dimensional models, are essential for understanding blood flow's role in bioheat transfer. One-dimensional models, as explained by [2], offer detailed insights by analysing heat transmission along specific tissue orientations, crucial for capturing localized temperature variations accurately [16]. Researchers combine these theoretical models with experimental data and clinical insights to push the boundaries of bioheat transfer research, leading to new thermal therapies and medical interventions. In conclusion, bioheat transfer integrates biology, medicine, and engineering to understand heat movement in living tissues. The Pennes bioheat equation is central to this field, and ongoing efforts aim to enhance temperature predictions and optimize thermal treatment strategies.

When it comes to bioheat transfer, it's very important to make sure that theory models are correct and can be used in real-world scenarios. Harold J. Pennes came up with the Pennes bioheat equation in 1948. It is one of the most important tools in bioheat transfer modelling because it adds blood flow effects to tissue temperature estimates to help us understand how biological tissues react to heat [7]. It is crucial to validate this equation through theoretical investigations to evaluate its accuracy in various physiological situations, improve thermal treatment procedures, and advance research on bioheat transfer. Moreover, one-dimensional modelling approaches, such as the simplified Pennes bioheat equation formulation, have been extensively explored to enhance predictive accuracy and validate their effectiveness in capturing localized temperature variations [5].

A thorough comprehension of heat transmission in living tissues is provided by these theoretical frameworks, along with improvements in one-dimensional modelling methods [6]. Research conducted by [16] has enhanced bioheat transfer models by examining one-dimensional modelling techniques for localized temperature variations. The validation process, especially analytical solutions, helps us learn more about bioheat transfer and makes it possible to create more effective thermal therapies for medical treatments, which eventually improves patient outcomes and safety in healthcare facilities.

In 1948, Pennes developed the Pennes bioheat equation [7], which has since become an essential model in the field of bioheat transfer. It explains the complex relationship between blood flow and tissue temperature. The analytical solutions to this equation have been studied by researchers to better understand the processes of heat transfer in biological systems [14]. Significant contributions by Weinbaum and Jiji challenged previous assumptions and highlighted the importance of counter current heat exchange in tissue, revolutionizing our perception of heat transfer in microvascular blood tissue [14]. Chato, Gupta & Johnson conducted studies that highlighted the significance of vascular architecture in heat transfer [3] [6]. They emphasised how blood vessels play a crucial role in affecting temperature distributions within biological systems.

Smith examined early studies on blood vascular and tissue interactions in bioheat transfer to improve our understanding of thermal dynamics in biological tissues [13]. Researchers have improved the analytical solutions of the Pennes bioheat equation to estimate blood and body temperature, advancing biomedical applications and healthcare. Bioheat transfer research combine early and modern findings to improve our understanding of heat transmission in living things.

Changes in body and blood temperature need to be understood and predicted for many biological uses, like thermal therapy and adapting to new environments. Extensive research has been conducted on models such as the Pennes bioheat equation and continuous models that account for blood flow effects in heat conduction equations. Pennes' 1948 bioheat equation has been foundational in this field [7]. Lemon’s found that blood artery size impacts heat transport, and recent computational developments have improved vascular models [11]. The studies conducted by Raaymakers and Chato investigated the process of heat transfer between blood arteries and the tissues around them, resulting in enhanced accuracy in prediction [3] [12]. According to Wissler, it is crucial to extensively validate theoretical models [17]. Additionally, Weinbaum proposed improvements to the Pennes perfusion term for better precision in areas with larger blood vessels [16]. These combined efforts enhance our ability to predict temperature profiles in biological tissues, benefiting therapies and environmental physiology.

2. Methodology

To derive the three-dimensional heat conduction equation from its one-dimensional form, we start with the one-dimensional equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \tag{1}$$

Here, $u(x,t)$ is the temperature distribution,α is the thermal diffusivity, and $\frac{\partial u}{\partial t}$ is the rate of temperature change over time. $\frac{\partial^2 u}{\partial x^2}$ represents the spatial temperature variation. Extending this to three dimensions, we consider heat transfer in the x, y, and z directions, leading to the three-dimensional heat conduction equation:

$$\rho c \frac{\partial T}{\partial t} = \nabla.(k\nabla T) + q \tag{2}$$

where, ρ is tissue density, *c* is tissue specific heat, *T* is temperature and *t* are time, *k* is the tissue thermal conductivity, *q* is the heat generation rate. To model biological tissues accurately, we add a term q_m for metabolic heat generation:

$$\rho c \frac{\partial T}{\partial t} = \nabla.(k\nabla T) + q_m \tag{3}$$

The impact of blood perfusion is included, considering local blood perfusion rate ω, blood density ρ_b, blood specific heat c_b, and the temperature difference between tissue and blood $(T - T_{b)}$:

$$\rho c \frac{\partial T}{\partial t} = \nabla.(k\nabla T) + q_m - \omega \rho_b c_b (T - T_b) \tag{4}$$

This equation, known as the Pennes Bioheat Equation, combines heat conduction, metabolic heat generation, and blood perfusion to describe temperature distribution in biological tissues. It is essential for understanding thermal interactions in living systems and developing thermal therapies.

To understand how body temperature is distributed, we use models like a simple cylinder or a three-part model for the torso, head, and limbs. The body loses heat through evaporation, convection, and radiation at the skin surface. The Pennes bioheat equation helps explain this by considering blood flow as a heat source. Metabolism generates heat within tissues, and blood flow can either heat or cool the tissue, depending on the temperature difference between the blood and the tissue. The temperature distribution in body tissue is governed by:

$$\rho c \frac{\partial T}{\partial t} = k_t \nabla^2 T_t + \rho c \omega (T_a - T_t) \tag{5}$$

This equation is solved with specific boundary and initial conditions. The skin boundary condition considers environmental temperature T_{air} and a convection coefficient h, accounting for convection, radiation, and evaporation. In steady state, the body maintains a balance between heat gain and loss. The total heat transfer from blood to tissue is:

$$\rho c Q_{blood-tissue,0} = \iiint_{body\ volume} \rho c \omega_{avg} (T_{a0} - \bar{T}_{t0}) V_{body} = 0 \tag{6}$$

During steady state, arterial blood temperature T_a equals the average tissue temperature $\bar{T}_t$. During changes like cooling or warming, the heat transfer from blood to tissue is given by:

$$Q_{blood-tissue,0} = \iiint_{body\ volume} \rho c \omega (T_{a0} - T_{t0}) dV_{body} \neq 0 \tag{7}$$

This equation helps predict body temperature changes under different conditions, aiding in clinical applications and thermal therapy. To manipulate body temperature during clinical applications, external heating or cooling of the blood can be employed using techniques like intravascular catheters or intravenous fluid infusion. Blood is treated as a lumped system, with an average volume of around 5 litters [17]. The energy change in the blood due to temperature variation can be expressed as:

$$\rho_{blood} c_{blood} V_{blood} [T_a(t + \Delta t) - T_a(t)]/\Delta t \approx \rho_{blood} c_{blood} V_{blood} \frac{dT_a}{dt} \tag{8}$$

The energy change in blood results from external heating or cooling ($Q_{ext)}$ and heat exchange with body tissue ($Q_{blood-tissue}$). The governing equation for blood temperature is:

$$\rho_{blood} c_{blood} V_{blood} \frac{dT_a}{dt} = Q_{ext}(T_a) - Q_{blood-tissue}(t) = Q_{ext}(T_a) - \rho c\bar{\omega} V_{body}(T_a - \bar{T}_t) \cdot \quad (9)$$

To accurately predict temperature distribution in both body tissue and blood, solve this equation along with the Pennes bioheat equation Eq. (5). Solving these equations together allows for a comprehensive understanding of the thermal dynamics within the human body, considering the interaction between blood and tissue temperatures.

3. Results and discussion

In thermal physiology and biomedical research, we simplify the analysis of temperature changes in body tissue and blood by focusing on key terms in the equations. The primary equation for tissue temperature Eq. (5) is simplified to:

$$\rho c \frac{\partial T}{\partial t} = \nabla.(k\nabla T) + q_m - \omega \rho_b c_b (T - T_b) \quad (10)$$

This focuses on the dynamic changes in tissue temperature $\rho c \frac{\partial T}{\partial t}$ and the convective heat transfer between blood and tissue $\omega \rho_b c_b (T - T_b)$. The energy change in blood due to external heating or cooling Q_{ext} and heat loss to body tissue $Q_{blood-tissue}$ is described by:

$$\rho_{blood} c_{blood} V_{blood} \frac{dT_a}{dt} = Q_{ext}(T_a) - \rho c\bar{\omega} V_{body}(T_a - \bar{T}_t) \quad (11)$$

This simplified model highlights the essential factors influencing temperature dynamics in blood and tissue during clinical procedures. The governing equation for blood temperature is a first-order ordinary differential equation. It can be solved numerically as most parameters in Equation (9) vary with time. We use the finite difference method to discretize the time derivative, employing the explicit method to avoid oscillations and maintain physical accuracy. The explicit forward-difference approximation is used as follows:

$$\rho_{blood} c_{blood} V_{blood} \frac{dT_a}{dt} = Q_{ext}(T_a) - \rho c\bar{\omega} V_{body}(T_a - \bar{T}_t) \quad (12)$$

Discretizing Eq. (9) with the explicit method:

$$T_a^{P+1} = T_a^P + \frac{\Delta t}{\rho_{blood} c_{blood} V_{blood}} \left[Q_{ext}(T_a) - \rho c\bar{\omega} V_{body}(T_a^P - \bar{T}_t^P) \right] \quad (13)$$

Where T_a^{P+1} represents the blood temperature at the next time step, T_a^P is the current blood temperature, and Δt is the time interval. This equation updates the blood temperature at each time step in the simulation. To avoid oscillations, the time step must satisfy:

$$1 - \frac{\rho c\bar{\omega} V_{body} \Delta t}{\rho_{blood} c_{blood} V_{blood}} \geq 0 \; or \; \Delta t \leq \frac{\rho_{blood} c_{blood} V_{blood}}{\rho c\bar{\omega} V_{body}} \quad (14)$$

The discretized equation for T_a using the implicit scheme can be written as,

$$\rho_{blood} c_{blood} V_{blood} (T_a^{P+1} - T_a^P) = \Delta t Q_{ext}(T_a^{P+1}) - \Delta t \, \rho c\bar{\omega} V_{body}(T_a^{P+1}) + \Delta t \, \rho c\bar{\omega} V_{body}(\bar{T}_t^P) \quad (15)$$

$$(T_a^{P+1}) = \frac{\rho_{blood} c_{blood} V_{blood}(T_a^P) + \Delta t \, \rho c\bar{\omega} V_{body}(\bar{T}_t^P)}{\rho_{blood} c_{blood} V_{blood} - \Delta t \{Q_{ext} + \rho c\bar{\omega} V_{body}\}} \quad (16)$$

In this example, we apply the thermal model to a blood cooling scenario using a small tube in the femoral vein, which has been clinically used to reduce temperatures in stroke or head-injured patients. The cooling device has a maximum capacity of 100 W. We consider a male weighing 81 kg with a body volume of 0.07421 m³. Two body geometries are tested, a simple cylinder (0.232 m in diameter, 1.8 m tall, surface area 1.312 m²) and a more realistic form with limbs, torso, neck, and head (surface area 1.8 m²). Using Mosteller's formula, the calculated body surface area is approximately 2.012 m², aligning closely with the detailed geometry.

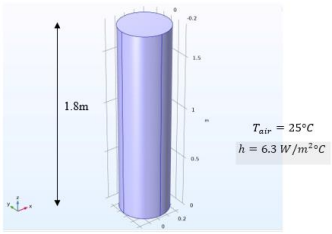

Figure 1 *Schematic Diagram of a Simplified Human Body Geometry*

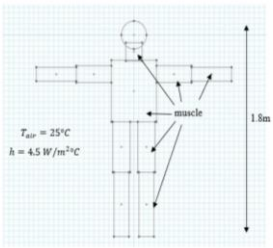

Figure 2 *Schematic Diagram of the Detailed Human Body Geometry*

The average stroke volume is 0.7 liters, and the heart rate is 75 beats per minute. The average blood perfusion rate is 6.773 ml/min per 100g of tissue or 0.001129 s^{-1}, based on a total body mass of 81 kg. For a simple one-compartment model, the average metabolic heat generation rate is estimated at 1250 W/m^3, based on a daily food intake of 2000 kCal/day. Clothing can be modelled as thermal resistance, contributing to the total heat transfer coefficient h. We assume $\omega, h,$ and T_{air} remain constant during cooling. The starting blood temperature is 37°C.

In the steady state, the convection heat transfer coefficient h is 4.7 W/m^2 °C for the detailed model and 6.3 W/m^2 °C for the simple model, corresponding to different body surface areas. The normal body temperature T_0 equals the blood temperature T_{a0}. For stable energy balance, biological heat generated inside the body must be dissipated through convection and radiation. Steady-state body temperatures range from 37.275°C in the brain to 34°C at the fingertips, with the highest temperature typically in the brain due to its high metabolic heat generation.

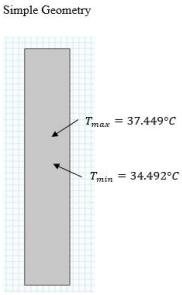

Figure 3 *Initial Steady State Temperature Contours of the Human Body in Simple Geometry*

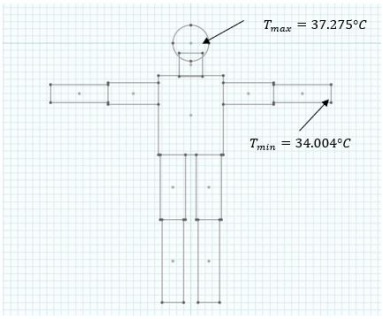

Figure 4 *Initial Steady State Temperature Contours of the Human Body in Detailed Geometry*

The cooling device has a capacity of around 100 W, so external cooling to the blood Q_{ext} is modelled as -100 W. The discretized equation for arterial temperature T_a using an explicit scheme is like Eq. (13),

$$T_a^{P+1} = T_a^P + \frac{\Delta t}{\rho_{blood} c_{blood} V_{blood}} \left[Q_{ext}(T_a) - \rho c \bar{\omega} V_{body} (T_a^P - \bar{T}_t^P) \right]$$

In this calculation, Δt is 60 seconds, which satisfies Eq. (14),

$$1 - \frac{\rho c \bar{\omega} V_{body} \Delta t}{\rho_{blood} c_{blood} V_{blood}} \geq 0 \; or \; \Delta t \leq \frac{\rho_{blood} c_{blood} V_{blood}}{\rho c \bar{\omega} V_{body}}$$

The discretized equation for T_a using the implicit scheme is like Eq. (16),

$$(T_a^{P+1}) = \frac{\rho_{blood} c_{blood} V_{blood} (T_a^P) + \Delta t \, \rho c \bar{\omega} V_{body} (\bar{T}_t^P)}{\rho_{blood} c_{blood} V_{blood} - \Delta t \{ Q_{ext} + \rho c \bar{\omega} V_{body} \}}$$

The time Δt is also 60 seconds. The arterial blood temperature over the first 20 minutes of cooling is shown in Figures 5 and 6. To minimize error in the time derivative approximation, the time step should be less than 60 seconds. Using the implicit approach, the arterial temperature can drop by up to 0.737°C in the first 20 minutes. In the detailed model, there's a significant initial temperature drop, followed by a stable decay rate of about 0.019°C/min. The arterial temperature is predicted to drop to 35.5°C after one hour and 34.4°C after two hours.

Table 1 lists the parameters used in the MATLAB script to simulate blood temperature changes during cooling using both implicit and explicit schemes.

Table 1: Parameters used in Both Models.

Parameter	Symbol	Value	Units
Density of blood	ρ_{blood}	1060	Kg/m^3
Specific heat of blood	C_{blood}	3800	J/(kg°C)
Volume of blood	V_{blood}	0.005	m^3
External cooling rate	Q_{ext}	-100	W
Volumetric average blood perfusion rate (simple geometry)	ω	0.001129	1/s
Volumetric average blood perfusion rate (detailed geometry)	ω_{detail}	0.0015	1/s
Initial arterial blood temperature	T_{a0}	37	$°C$
Time step	Δt	60	s
Total simulation time	t_{total}	20 * 60	s
Number of steps	n_{steps}	$t_{total}/\Delta t$	-
Specific heat of body tissue	ρ	3500	J/(kg°C)
Density of body tissue	C	1000	Kg/m^3
Volume of body tissue	V_{body}	0.1	m^3

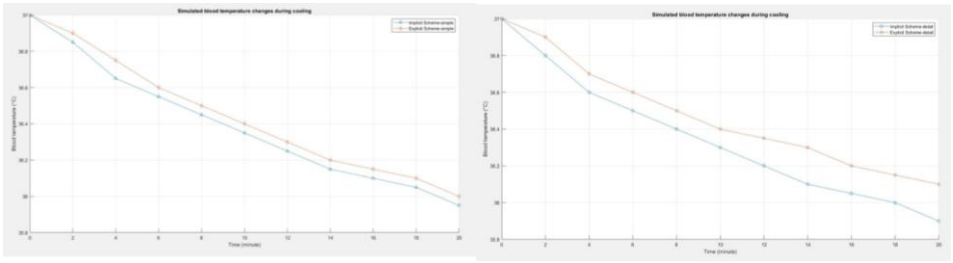

Figure 5 Simulated Blood Temperature Changes During the Cooling using Both Implicit and Explicit Schemes. (a) the Simple Model and (b) the Detailed Model.

In whole-body cooling for stroke or head injury patients, physicians focus on body tissue temperature. Figure 6 shows the volumetric average body temperature (T_{avg}), weighted average body temperature ($\bar{T}_t$), maximum tissue temperature, and minimum skin surface temperature. The difference between T_{avg} and $\bar{T}_t$, is due to their definitions. After 20 minutes of cooling, tissue temperatures drop by 0.3 to 0.5°C, almost linearly. Skin temperature drops more slowly, by 0.2°C per 20 minutes. Figure 7 shows that blood temperature in the detailed model drops quickly at first (about 0.14°C/min) and then

stabilizes after 20 minutes. Cooling the entire body, measured by T_{avg}, starts slowly and gradually catches up, possibly due to the body's inertia in reacting to blood cooling. After the initial fluctuation, cooling rates of all temperatures stabilize at around 0.019°C/min (1.15°C/hour). In the simple model, cooling rates converge to about 0.018°C/min after 10 minutes. This cooling method can induce mild body hypothermia (34°C) within three hours.

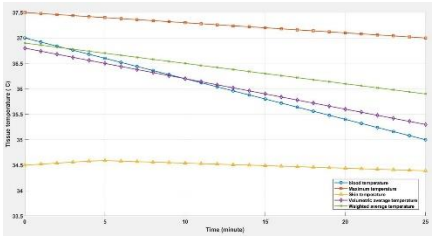

Figure 6 Temperature Decays During the Cooling Process Using the Detailed Geometry and Implicit Scheme.

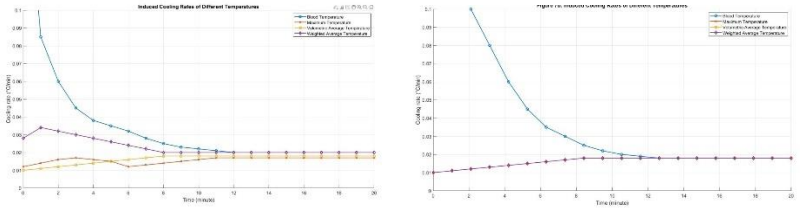

Figure 7 Induced Cooling Rates of the Blood Temperature, the Maximum Temperature, the Volumetric Average Temperature, and the Weighted Average Temperature. (A) the Detailed Model and (B) the Simple Model.

In Eq. (13), if blood doesn't absorb heat from tissue, the first term indicates external cooling rate, expected around 18.9°C/hour, significantly higher than actual cooling rate due to device cooling entire body. The second term, representing blood-tissue thermal exchange, depends on blood-tissue temperature difference. Initially zero, this difference stabilizes at -2.5°C about 20 minutes into cooling. Once stabilized, Eq. (13) right side becomes constant due to steady assumptions.

Figure 9 shows how body temperature changes with simplified body geometry using implicit scheme (60-second time step). Despite different geometries, cooling rates for volumetric average body temperature (T_{avg}), are similar. Simple model lack's ability to show actual temperature changes during cooling.

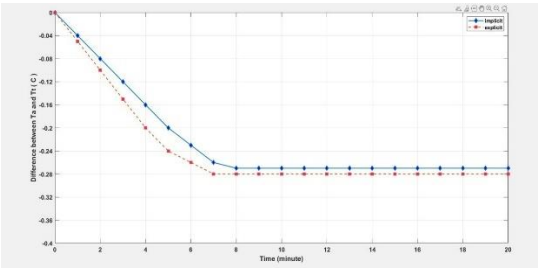

Figure 8 Difference Between the Blood Temperature and the Weighted Average Body Temperature During Cooling

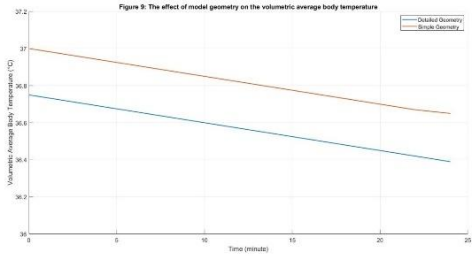

Figure 9 The Effect of Model Geometry on the Volumetric Average Body Temperature

Conclusion

In conclusion, this study presents a theoretical model for simulating thermal interactions between blood and tissue during blood cooling and rewarming processes in the human body. The model, integrating the Pennes bioheat equation with a blood heat transfer equation, accurately predicts body and blood temperature changes crucial for clinical applications like managing systemic hypothermia or hyperthermia. While the model demonstrates reliability, limitations include simplified body geometry and assumptions about physiological parameters, potentially impacting real-world accuracy. Future research could address these limitations to further enhance temperature management strategies in medical treatments and physiological studies.

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