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Geometric Brownian Motion For Groundwater Level Prediction

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Abstract

This study addresses the critical issue of declining groundwater levels exacerbated by unsustainable abstraction practices during dry spells. It proposes a robust model using Geometric Brownian Motion (GBM), a stochastic differential equation adept at capturing the random and continuous nature of groundwater dynamics. The objectives focus on determining an appropriate model for predicting groundwater fluctuations in Malaysia and evaluating its accuracy using Root Mean Square Error (RMSE). By estimating model parameters like drift and volatility from groundwater data, the study demonstrates the model's efficacy in accurately forecasting future groundwater levels. Methodologically, it employs the Augmented Dickey-Fuller (ADF) test for stationarity, normality tests, and Maximum Likelihood Estimation for parameter estimation, alongside the Kolmogorov-Smirnov test for distribution fitting. Statistical software analysis validates the GBM model against traditional methods, showing strong predictive accuracy with Mean Absolute Percentage Errors (MAPE) of 5.91% for the Terengganu dataset and 0.15% for the Machap Dam dataset. This research highlights GBM's potential as a reliable tool not only for forecasting price fluctuations but also for managing groundwater dynamics characterized by high volatility. The results provide valuable insights for managers to use the information for managing groundwater management practices.

Keywords: Groundwater Level; Stochastic Differential Equation; Geometric Brownian Motion; Forecasting; MAPE.

Introduction

Groundwater, as a vital component of the hydrological cycle, refers to the water stored beneath the Earth's surface within porous rock formations called aquifers. It represents a critical source of freshwater globally, particularly in regions where surface water availability is limited. Groundwater originates from precipitation that percolates into the ground, eventually accumulating within underground reservoirs. This natural resource sustains various essential functions, including supporting ecosystems, supplying drinking water for communities, irrigating agricultural lands, and facilitating industrial processes. The dynamic nature of groundwater is characterized by its ability to be recharged during periods of rainfall.

This study addresses the pressing challenges of potable water supply in major Malaysian cities exacerbated by population growth, industrialization, surface water pollution, and recurring droughts. Particularly in Selangor, Kuala Lumpur, Johor Bahru, and Pulau Pinang, the demand for clean water is acute [1]. Groundwater has emerged as a crucial resource during dry spells, offering a vital alternative in regions like Melaka, Selangor, and Sarawak [3]. Groundwater, a critical freshwater source for global populations, is utilized for domestic, agricultural, and industrial purposes, especially vital in arid and semi-arid regions where surface water is scarce [4]. Recent studies highlight a concerning decline in groundwater levels due to unsustainable extraction practices, akin to issues faced in Malaysia and other Asian countries. This research aims to assess water resources comprehensively through mathematical modeling to develop an accurate stochastic groundwater prediction model, crucial for sustainable water management strategies.

Literature Review

Groundwater Level Fluctuations

Groundwater fluctuations, a critical component of hydrological systems, vary significantly across different regions due to complex interactions of climate, geology, and human activities. Studies like [6] highlight the impact of declining groundwater levels, particularly due to limited recharge and extensive pumping, on agricultural sustainability, exemplified by the Ogallala Aquifer. [7] discuss similar challenges in arid regions, emphasizing over-pumping and low recharge rates as causes of groundwater depletion. In Malaysia, despite a tropical climate with substantial rainfall, changing climatic conditions have led to irregular precipitation patterns, affecting water resources and agricultural sustainability [10].

Review on Groundwater Fluctuations Forecasting Research

By reviewing the literature of groundwater level forecasting, it can be observed that researchers commonly addressed through artificial intelligence (AI) models, particularly machine learning techniques such as Support Vector Machines, Random Forests, and Gradient Boosting, alongside traditional methods like Autoregressive Integrated Moving Average (ARIMA). Researchers, such as [2] and [19] , have applied these models to predict groundwater levels based on factors like rainfall and evapotranspiration. However, the use of stochastic differential equation (SDE) models, common in financial forecasting, has been scarcely explored in groundwater prediction research despite its potential to offer novel insights and potentially enhance prediction accuracy. [5] demonstrated the applicability of geometric Brownian motion for short-term stock price forecasting, suggesting an opportunity for further exploration of stochastic models in hydrological forecasting, which could significantly advance the field of groundwater fluctuation prediction.

Stochastic Differential Equation

A stochastic differential equation (SDE) is a differential equation containing one or multiple stochastic components that can be used to derive a solution [11]. SDE are usually selected to model systems with large random components, applicable across diverse fields including quantitative finance, meteorology, and hydrology [12]. [13] extensively reviewed the use of SDEs in hydrology, highlighting fundamental concepts such as Markov processes and Ito's calculus. This approach involves selecting suitable model structures, parameter estimation, and predictive modeling, offering a robust framework for enhancing accuracy in groundwater level forecasting. By leveraging SDEs for short-term prediction of groundwater fluctuations, this research aims to build upon existing hydrological forecasting methodologies and contribute to advancing predictive capabilities in water resource management.

Review on Groundwater Forecasting Using SDE

Stochastic differential equation is widely used to describe processes in seismology/seismic, groundwater, thermal, etc. Groundwater modeling for flow equations and forecasting water level heights assume the controlling physical parameters such as hydraulic conductivity, sources such as recharge and boundary conditions subject to a degree of error and uncertainty due to inaccurate measurements or inherent uncertainty in the parameter. Stochastic modeling helps in evaluating objectively the accuracy of groundwater predictions [16].

Methodology

Description of Data

In this study, the data that was used is the monthly water table depth data extracted from Musa G. Abdullahi's paper which was published in the Journal of Remote Sensing & GIS in April 2015. The range of the data are from January 2001 until December 2012.

Kruskal-Wallis Test

The Kruskal–Wallis test is a statistical test used to compare two or more groups for a continuous or discrete variable. It is a non-parametric test, meaning that it assumes no particular distribution of your data and is analogous to the one-way analysis of variance (ANOVA). In the Kruskal-Wallis test, the test variable H is calculated. The H value corresponds to the χ^2 value.

Augmented Dickey-Fuller Test

It is commonly employed to determine whether a time series is stationary or non-stationary. This test is particularly useful in detecting trends in time series data.

Hypothesis:

H_0: The time series has a unit root, indicating it is non-stationary.

H_1: The time series does not have a unit root, indicating it is stationary.

If the test statistic is less than the critical values, the null hypothesis is rejected, suggesting that the time series is stationary.

Groundwater Level Log Return

The formula of logarithm return for water level depth is defined as following:

$$D_t = \ln\left(\frac{W_t}{W_{t-1}}\right) \tag{1}$$

where:

D_t: Water level logarithm return at time t

W_t: Water level at time t

W_{t-1}: Water level at time t-1

Normality test

The Anderson-Darling test is used to test normally or not normal distributed data.

H_0: The data follows the normal distribution

H_1: The data do not follow the normal distribution

Statistical test:

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln F(D_t) + \ln(1 - F(D_{n-i+1})] \tag{2}$$

where:

n: sample size

i: ith sample when the data is sorted in ascending order

$F(x)$: cumulative distribution function for the specified distribution

Maximum Likelihood Estimation

The computation of the parameters was carried out using Microsoft Excel. We computed monthly groundwater level returns from January, 2001 to December, 2012, and we adopted the same relation used in (Iversen et al., 2016) which is given as

$$D_i = \frac{G_t - G_{t-1}}{G_{t-1}} \tag{3}$$

where D_t denotes the water level returns at time t, W_t and W_{t-1} a represent water level at time, t and time $t-1$ respectively. Using Equation (3), we computed the value of the drift (μ) using Equation (4) which gives as

$$\mu = \frac{\sum_{t=2}^{60}(D_t)}{59} \tag{4}$$

where μ is the drift, D_t as defined above, n is the number of groundwater level returns. However, with known value of drift, we need to compute the volatility value with the formula in Equation (5)

$$\sigma_d^2 = \sum_{t=2}^{n} \frac{(D_t - \overline{D})^2}{n-1} \tag{5}$$

where σ_d^2denotes volatility and $\overline{D}$ a represents the groundwater level return.

Geometric Brownian Motion

Geometric Brownian motion (GBM) is a fundamental stochastic process used to describe commodity price changes. In this research, this method is used to describe the changes in groundwater levels as a random walk with a drift and volatility component, reflecting the exponential nature of groundwater fluctuations. The GBM equation for forecasting the groundwater level at time *t* can be expressed as:

$$G_t = G_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma w_i} \tag{6}$$

where:

G_t : actual groundwater level at time t

G_{t-1} : forecast groundwater level at time $t-1$

μ : drift value

σ : volatility value

Mean Absolute Percentage Error (MAPE)

The model's effectiveness is assessed using the Mean Absolute Percentage Error (MAPE) metric, comparing the actual groundwater levels with the predicted ones. Equation of MAPE as follows:

$$MAPE = \frac{1}{N}\sum_{i=1}^{m}\left|\frac{A_i - F_i}{A_i}\right| \; x\ 100\% \tag{7}$$

where:

A_i : actual groundwater level at time i

F_i : forecast groundwater level at time i

N : number of simulations

Results and discussion

Data Collection

The historical monthly water table depth data was extracted from Musa G. Abdullahi's paper which was published in the Journal of Remote Sensing & GIS in April 2015. Exploring and investigating the appropriate model and predicting groundwater level fluctuations in Malaysia using the stochastic differential equation is the main aim of this research.

Kruskal-Wallis Test

The output for Kruskal-Wallis test by using Minitab shows a smaller *p*-value indicating the existence of seasonality as there is enough evidence that the medians for the months data are not the same at the chosen significance level at 0.05.

Augmented Dickey-Fuller Test

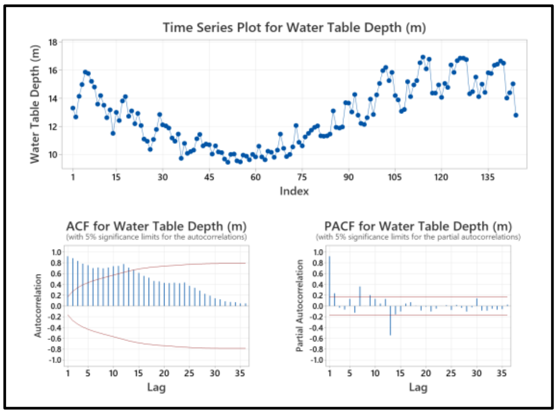


Figure 1 Output For Augmented Dickey-Fuller Test

The data is non stationary at the significance level of 0.05. Based on this result, the stochastic differential equations approach is considered in this research study for the groundwater level fluctuations prediction. Stochastic differential equations are able to capture the random and continuous nature of the underlying processes, making them suitable for modeling complex and non-stationary time series data. This might enhance the model's ability to handle the specific characteristics of the dataset and improve forecasting accuracy.

Normality test

At this stage, a test of normality is performed from January 2001 to December 2012, which has been obtained from the previous stage. A normality test is performed to determine whether the logarithm return of water level is normally distributed or not. Here is the step to test the normality of water level logarithm returns.

$$AD = -143 - \frac{1}{143}\sum_{i=1}^{n}(2i-1)[lnF(D_t) + ln(1 - F(D_{n-i+1})]$$

$$= 0.3262$$

$$AD^* = AD(1 + \frac{0.75}{59} + \frac{2.25}{59^2})$$

$$= 0.3279$$

The value of p-value = 0.531 > 0.05, fail to reject H_0 , then we can conclude that the logarithm return of water level data is normally distributed. Histogram of logarithm return is present in the Figure 3:

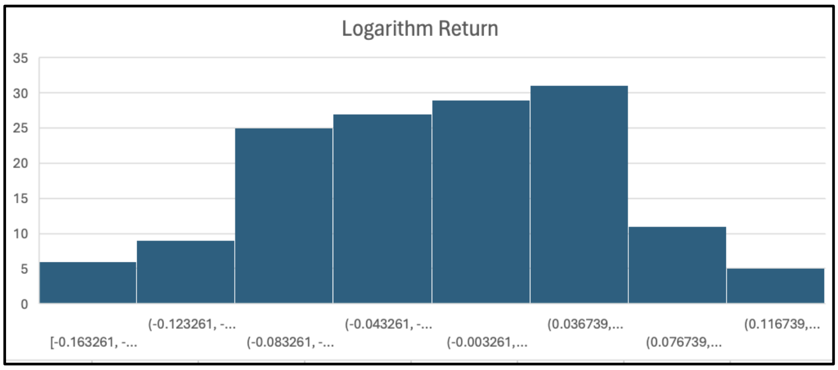


Figure 3 Histogram of Water Level Return

Calculation of Log Return

$$D_2 = ln(\frac{W_2}{W_1}) = ln(\frac{12.68}{13.31}) = -0.048816$$

$$D_{144} = ln(\frac{W_{144}}{W_{143}}) = ln(\frac{12.79}{15.02}) = -0.160379$$

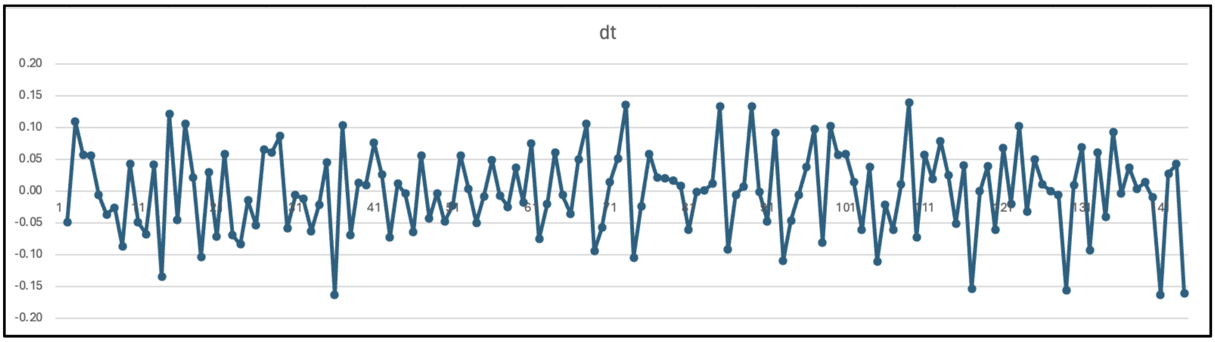


Figure 4 Logarithm Return of Water Level versus Time

Parameter Estimation

The estimate of volatility parameters and drift using maximum likelihood estimation is done at this step. The values of the drift and volatility parameters are constant, and this is the value that had been used to forecast groundwater level in the future. Calculating the average log discharge and standard deviation of log discharge is the first step in determining the value of volatility. By using the results of the water level log discharge computation in the preceding calculation, the calculation of average log discharge, μ using equation (4) are presented below:

$$\mu = \frac{\sum_{t=2}^{60}(D_t)}{59} \tag{8}$$

$$= \frac{D_2 + D_3 + D_4 + \ldots + D_{143} + D_{144}}{143}$$

$$= \frac{-0.048816 + 0.109265 + 0.057795 + \ldots + 0.042387 + (-0.160379)}{143}$$

$$= -0.000278$$

After the calculation of logarithm return has been done, the standard deviation of logarithm return needs to be calculated. By using equation (5), the calculation of standard deviation of logarithm return have been done and presented as below:

$$\sigma_d^2 = \sum_{t=2}^{n} \frac{(D_t - \underline{D})^2}{n-1} \tag{9}$$

$$= \frac{(-0.048816 - (-0.000278))^2 + (0.109265 - (-0.000278))^2 + \ldots + (0.042387 - (-0.000278))^2}{142}$$

$$= 0.004261$$

$$\sigma_d = 0.065277$$

Then, the value of $\hat{\mu}$ and $\hat{\sigma}$ are as shown below:

$$\hat{\mu} = -0.000278$$

$$\hat{\sigma} = 0.065277$$

After the calculation, the RMSE value is 0.96, it can be concluded that this gives a better estimate of the data.

Data Prediction

Forecasting groundwater level using geometric Brownian motion (GBM) encompasses utilizing the stochastic process to estimate future groundwater level fluctuations based on historical data with the assumption that the fluctuation returns follow a normal distribution. We can compute the volatility (σ) and the drift coefficient (μ) from the data using Equation (4) and (5). The obtained parameter values are then substituted into Equation (6), with the initial water table depth set as G(0). Future water table depth is simulated using the calculated drift and volatility. This involves generating random numbers from a standard normal distribution to serve as the random part of the equation, the random numbers are then multiplied by the square root of the time increment (dt) and the volatility to simulate the random increment (dw_i).

$$G_t = G_{t-1} e^{(\hat{\mu} - \frac{1}{2}\hat{\sigma}^2)dt + \hat{\sigma}\varepsilon\sqrt{dt}}$$

$$G_1 = G_0 e^{(-0.000278 - \frac{1}{2}(0.065277)^2)dt + 0.065277\varepsilon\sqrt{dt}}$$

$$G_1 = 13.38$$

.

$$G_{144} = G_{143}e^{\left(-0.000278-\frac{1}{2}(0.065277)^2\right)dt + 0.065277\varepsilon\sqrt{dt}}$$

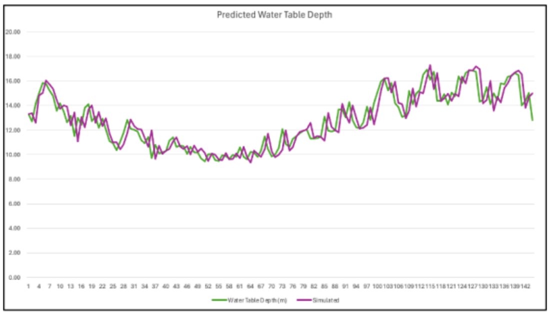


Figure 5 Graph of the actual and predicted water table depth

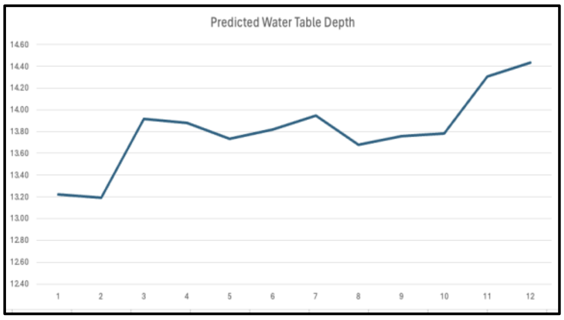


Figure 6 Graph of predicted water table depth from January 2013 to December 2013

Figure 5 shows a comparison of the predicted groundwater level and the actual groundwater level. This shows how well the model can capture the trend in the data. We further deployed the model to predict the groundwater level fluctuations for an additional twelve (12) months, ranging from January 2013 to December 2013. The predicted values for 12 months were shown in Table 1, while Figure 6 shows the graph of the predicted water table depth until December 2013.

Table 1: The Predicted Water Table Depth for 12 Months

Month	Predicted Value	Month	Predicted Value
January, 2013	13.22	July, 2013	13.95
February, 2013	13.19	August, 2013	13.68
March, 2013	13.91	September, 2013	13.76
April, 2013	13.88	October, 2013	13.78
May, 2013	13.74	November, 2013	14.31
June, 2013	13.82	December, 2013	14.43

Data Prediction

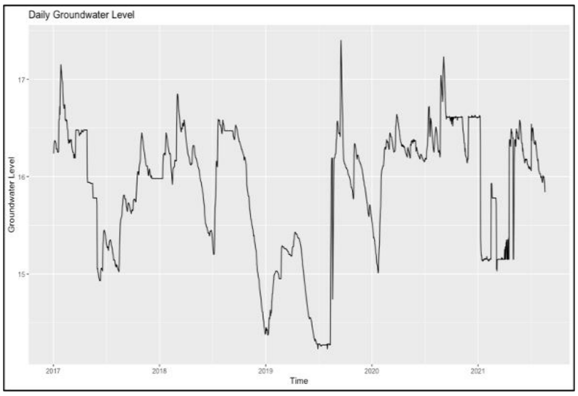
After the results of groundwater level predictions, the Mean Absolute Percentage Error (MAPE) value prediction is counted at a later stage using Equation (7) to determine the geometric Brownian motion model's level of accuracy in forecasting groundwater levels.

$$MAPE = \frac{1}{N}\sum_{i=1}^{m}\left|\frac{A_i - F_i}{A_i}\right| \, x\, 100\% = \frac{1}{12}(0.709291)\, x\, 100\% = 5.91\%$$

The MAPE value of the predicting portion in GBM is satisfying since it is 5.91%, which is extremely accurate according to the table of accuracy.

Data Analysis of Groundwater Level Data, Machap Dam

This research utilized an additional dataset which is the hourly groundwater level data from Machap Dam, spanning January 2017 to December 2021, sourced from the Department of Irrigation and Drainage Malaysia. Daily groundwater level abstraction was analyzed and visualized using R software.



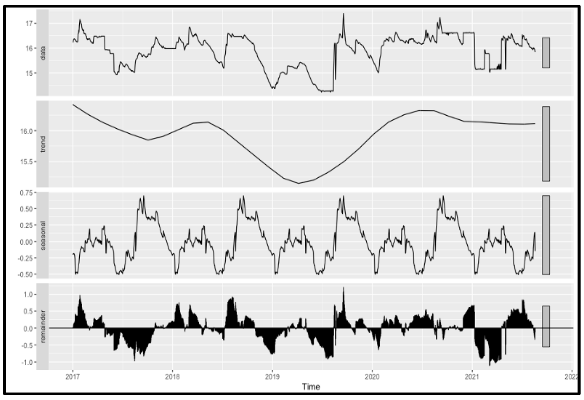


Figure 7 Time Series Plot for GWL Data, Machap Dam

Figure 8 Seasonal Decomposition Plot

The seasonal decomposition plot, particularly the middle panel representing the seasonal component, shows minimal variation or pattern, indicating no distinct recurring cycles or trends across different months or periods. Additionally, the Kruskal-Wallis test's non-significant p-value reinforces the conclusion that there are no systematic differences or seasonal effects observed in groundwater levels over time. This implies that the groundwater levels at Machap Dam do not exhibit seasonal variations that would require specific seasonal adjustments or considerations in analysis and management.

Parameter Estimation

Then, the value of $\hat{\mu}$ and $\hat{\sigma}$ are as shown below:

$$\hat{\mu} = -0.000015$$

$$\hat{\sigma} = 0.003852$$

Thus, the equation become

$$G_t = G_{t-1}e^{\left(-0.000015-\frac{1}{2}(0.003852)^2\right)dt\,+\,0.003852\varepsilon\sqrt{dt}}$$

The simulated data for the first 10 periods is shown in Table 2. After the calculation, the RMSE value is 0.34, it can be concluded that this gives a better estimate of the data.

Data Prediction

Table 2: Predicted GWL Data for the First 10 Period

Period	Predicted Value	Period	Predicted Value
1	16.24	6	16.36
2	16.24	7	16.38
3	16.24	8	16.35
4	16.30	9	16.38
5	16.34	10	16.34

Figure 9 below also shows the plot for actual and predicted water table depth while Figure 10 shows the graph of forecasted water table depth from January 2013 to December 2013.

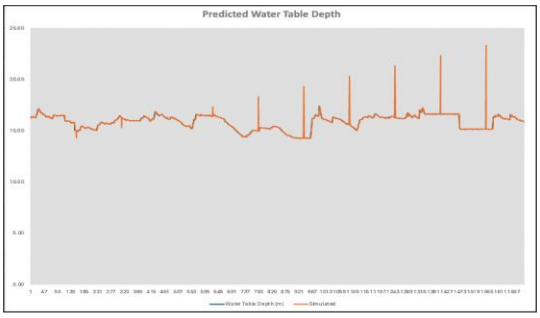


Figure 9 Graph of the actual and predicted water table depth

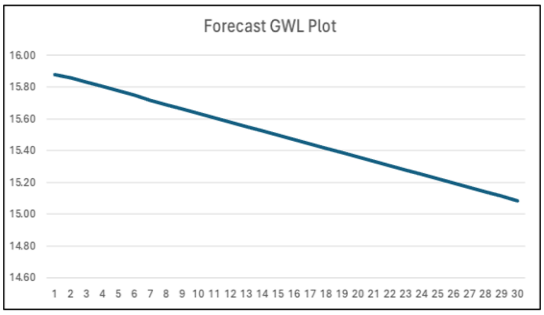


Figure 10 Graph of predicted water table depth from January 2013 to December 2013

After the results of groundwater level predictions, the Mean Absolute Percentage Error (MAPE) value prediction is counted at a later stage using Equation (7) to determine the GBM model's level of accuracy in forecasting groundwater levels for this dataset.

$$MAPE = \frac{1}{N}\sum_{i=1}^{m}\left|\frac{A_i - F_i}{A_i}\right| \, x \, 100\% = \frac{1}{30}(0.047911) \, x \, 100\% = 0.15\%$$

The MAPE value of the predicting portion in this model is satisfying since it is 0.15%, which is extremely accurate according to the table of accuracy.

Conclusion

In conclusion, this study used the Geometric Brownian Motion (GBM) model to forecast groundwater level fluctuations in two datasets: Terengganu, with significant seasonality, and Machap Dam, with stationary behavior. The model's performance was evaluated using Mean Absolute Percentage Error (MAPE), showing high predictive accuracy. For Terengganu, the GBM model achieved a MAPE of

5.91%, effectively forecasting future levels despite seasonal variability. For Machap Dam, the MAPE was 0.15%, demonstrating the model's robustness in stable conditions. These results highlight the GBM model's reliability in diverse groundwater dynamics, providing valuable insights for water resource management and supporting sustainable environmental practices.

Acknowledgement

I would like to express my sincere gratitude to Dr. Arifah Bahar for her exceptional guidance and support during my research, which focused on exploring and predicting groundwater level fluctuations in Malaysia using stochastic approaches. Her invaluable feedback and advice significantly contributed to the success of this study. I also thank my family and friends for their continuous encouragement. I acknowledge Musa G. Abdullahi's paper provides the historical water table depth data used in this research, which aims to advance our understanding of hydrological processes and groundwater dynamics. Lastly, my appreciation extends to the Department of Irrigation and Drainage that generously provided the groundwater level data, as their contribution has been indispensable to the completion of this research endeavor.

References

[1] Saimy, I. S., & Raji, F. (2015). Applications and sustainability in groundwater abstraction in Malaysia. Jurnal Teknologi (Sciences & Engineering), 73(5), 39–45.

[2] Yan, Q., & Ma, C. (2016). Application of integrated ARIMA and RBF network for groundwater level forecasting. Environmental Earth Sciences, 75.

[3] Mohamed, A. F., Yaacob, W. Z. W., Taha, M. R., & Samsudin, A. R. (2009). Groundwater and soil vulnerability in the Langat Basin Malaysia. European Journal of Scientific Research, 27(4), 628–635.

[4] Siebert, S., Burke, J., Faures, J. M., Frenken, K., Hoogeveen, J., Döll, P., & Portmann, F. T. (2010). Groundwater use for irrigation – a global inventory. Hydrol. Earth Syst. Sci., 14, 1863–1880.

[5] Abidin, S., & Jaffar, M. M. (2012). A review on geometric Brownian motion in forecasting the share prices in Bursa Malaysia. World Applied Sciences Journal, 17, 87-93.

[6] Alley, W. M., Reilly, T. E., & Franke, O. L. (2002). Sustainability of groundwater resources.Science, 296 (5575), 1980-1982.

[7] Foster, S. S. D., & Chilton, P. J. (2003). Groundwater in the Environment: An Introduction.CRC Press.

[8] Bjerregård, M. B., Møller, J. K., Brok, N. B., Madsen, H., & Christiansen, L. E. (2022). Probabilistic forecasting of rainfall response in a Danish stormwater tunnel. J. Hydrology, 612.

[9] Ghahramani, S. (2005). Fundamentals of Probability with Stochastic Processes (3rd ed.). Pearson.

[10] Kayode, J. S., Arifin, M. H., Hussin, A., & Roslan, N. (2019). Engineering site characterization using multi-electrode electrical resistivity tomography. Sains Malays., 48(5), 945-963.

[11] Iversen, E. B., Morales, J. M., Møller, J. K., & Madsen, H. (2016). Short-term probabilistic forecasting of wind speed using stochastic differential equations. Int. J. Forecast., 32(3), 981–990.

[12] Li, H. (2022). Short-term wind power prediction via spatial temporal analysis and deep residual networks. Front. Energy Res., 10.

[13] Bodo, B. A., Thompson, M. E., & Unny, T. E. (1987). A review on stochastic differential equations for applications in hydrology. Journal of Stochastic Hydrology and Hydraulics, 1, 81–100.

[14] Oh, Y.-Y., Hamm, S.-Y., Ha, K., Yoon, H., & Kim, G.-B. (2016). Analytical approach for evaluating the effects of river barrages on river aquifer interactions. Hydrological Processes, 30, 3932–3948.

[15] Oh, Y.-Y., Yun, S.-T., Yu, S., & Hamm, S.-Y. (2017). The combined use of dynamic factor analysis and wavelet analysis to evaluate latent factors controlling complex groundwater level fluctuations in a riverside alluvial aquifer. Journal of Hydrology, 555, 938–955.

[16] Srivastava, K. (2019). Stochastic Modeling in Earth Sciences. Journal of Geological Society of India, 93, 255–256.
[17] Singh, A., Bhardwaj, S., Kumar, S., & Kumar, N. (2018). A review: Groundwater level forecasting using artificial neural network. Proceedings of the Conference, 2433–2436.
[18] De Souza Groppo, G., Costa, M. A., & Libânio, M. (2019). Predicting water demand: A review of the methods employed and future possibilities. Water Supply, 19, 2179–2198.
[19] Singh, A., Patel, S., Bhadania, V., & Kumar, V. (2018). AutoML-GWL: Automated machine learning model for the prediction of groundwater level. Engineering Applications of Artificial Intelligence.
[20] Abdullah, M. G., & Garba, I. (2015). Effect of rainfall on groundwater level fluctuation in Terengganu, Malaysia. Journal of Remote Sensing & GIS.
[21] Lamontagne, S., Taylor, A. R., Cook, P. G., Crosbie, R. S., Brownbill, R., Williams, R. M., & Brunner, P. (2012). Field assessment of surface water-groundwater connectivity in a semi-arid river basin (Murray-Darling, Australia). Hydrological Processes, 28, 1561–1572.
[22] Brédy, J., Gallichand, J., Celicourt, P., & Gumiere, S. J. (2020). Water table depth forecasting in cranberry fields using two decision-tree-modeling approaches. Agricultural Water Management, 233, 106090.
[23] Rahman, A. S., Hosono, T., Quilty, J. M., Das, J., & Basak, A. (2020). Multiscale groundwater level forecasting: coupling new machine learning approaches with wavelet transforms. Advances in Water Resources, 141, 103595..
[24] Ibrahem Ahmed Osman, A., Najah Ahmed, A., Chow, M. F., Feng Huang, Y., & ElShafie, A. (2021). Extreme gradient boosting (XGBoost) model to predict the groundwater levels in Selangor, Malaysia. Ain Shams Engineering Journal, 12(2), 1545-1556.
[25] Iversen, E. B., Morales, J. M., Moller, J. K., & Madsen, H. (2016). Short-term probabilistic forecasting of wind speed using stochastic differential equations. International Journal of Forecasting, 32(3), 981-990.
[26] Hasan, M. R., Mostafa, M. G., & Matin, I. (2013). Effect of rainfall on groundwater level fluctuations in Chapai Nawabganj District. International Journal of Engineering Research and Technology, 2(4), 2800-2807.