Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia
Vol. 23, 2024, page 73-81

Electromagnetic Blood Flow Effects Through Stenosed Artery Using Caputo-Fabrizio Fractional Derivatives

Sharvin Sundran, Dzuliana Fatin Jamil
Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia
Corresponding author: dzulianafatin@utm.my

Abstract

This study investigates a fractional order time derivatives model of electromagnetic blood flow with magnetic particles through stenosed artery in the presence of electric and magnetic field. The fluid was driven by pressure gradient and perpendicular magnetic field. The governing fractional differential equations were expressed using the Caputo-Fabrizio fractional derivative without singular kernel. The exact analytical solutions for the blood flow and magnetic particles velocities were obtained by means of Laplace and finite Hankel transform. Using the help of Mathcad software, a graphical analysis was produced to model the effects of fractional parameters, Hartmann number and Reynolds number on both velocities. The findings show that while increasing the Hartmann number decreases magnetic particle velocities due to the rising resistive force, the fractional parameter increases both blood flow and magnetic particle velocities. Blood viscosity is reduced by higher Reynolds numbers, which enhance flow, while by increasing Hartmann number slows down the velocity of blood flow. Drag force causes blood flow velocity to exceed magnetic particle velocity at zero electric field. These results bear the significant applications in the diagnosis and therapeutic treatment of some medical problems.

Keywords: blood flow, stenosed artery, fractional derivatives, electromagnetic field

1. Introduction

Over the decades blood flow modeling has found extensive use in enhancing our understanding of the signals associated with different diseases. This in turn helps us improve existing treatments and develop ones. A computational model of blood flow plays a role not in diagnosing clinical diseases but also as a vital part of modeling complex structures. Studies on blood flow often focus on different aspects, such as the effects of external forces like electromagnetic fields, the behavior of magnetic particles mixed with blood, and the impact of factors like viscosity and temperature on blood flow patterns.In a study conducted by Sanyal, Das & Debnath [1] they analyzed the characteristics of blood flow by utilizing a model that simulated a circular tube. This simulation also considered body acceleration and the influence of a magnetic field. The viscous elastic blood flow through a porous media in a cylindrical artery was investigated by Mohan [2]. In his view, the magnetic field had an impact on axial velocity. The studies on blood viscosity and blood flow in the stenosed arteries have gained significant focus in the last few years due to their relevance in the analysis of human circulatory system. Shah and Kumar [3] analyzed the mathematical analysis of blood flow with nanoparticles in suspension through a tapered artery that contains a clot to investigate the hemodynamics of blood flow during stenosis. Furthermore, Ponalagusamy and Manchi [4] in their recent work numerically analyzed a two-fluid model to study the flow of K–L fluid in a stenosed artery having a porous wall to understand the effects of constriction in blood vessels. Hamza et al. [5] presented the Carreau model to analyze the impact of several hemodynamic factors such as velocity, vorticity, strain rate, and WSS on stenosed and healthy arteries to identify the location of plaques and clarify the flow conditions in the stenotic zone. Moreover, Shaikh et al. [6] conducted a numerical comparison of segments of artery before and after stenosis in physiological and pathological scenarios to highlight the natural flow behavior of blood and the pathophysiology of stenosed arteries. These works enhance the current knowledge on blood flow through stenosed arteries and help in the development of medical and engineering solutions.

Electromagnetic blood flow appears to be a very promising new technology that has severalpossible applications in medicine and engineering. It is also the use of electrical and magnetic fields to control blood flow. Surgical procedures including cancer (tumor) surgeries have used electromagnetic blood flow. The artificial blood velocity was evaluated experimentally at various magnetic field strengths, and it was discovered that the velocities of blood and magnetic particles decreased dramatically in the presence of a magnetic field [7]. Although the behaviors of magnetic particles and blood are quite similar, the magnetic particle velocity is lower than the blood velocity due to drag and other resistive forces [8]. Omamoke et al. [9] focused on the consequences of thermal radiation and heat source in combination with magnetohydrodynamic blood flow, emphasizing the intricate connection between thermal effects, magnetic fields, and blood flow. Moreover, Yang et al. [10] proposed a multi-electrode electromagnetic flow detection method for non-invasive blood vessel measurement, indicating the potential for electromagnetic techniques in clinical applications. There have also been numerical studies on how magnetic fields affect blood flow. Selvi et al. [11] investigated the influence of electromagnetic fields and thermal radiation on pulsatile blood flow with nanoparticles in a constricted porous artery. They also studied the magnetic field effects on blood flow with Sisko fluid and Titanium magneto-nanoparticles.

Fractional calculus is a powerful tool for modeling electromagnetic blood flow problems, as blood is a viscoelastic fluid with memory effects. According to Caputo [12], in practical math problems, fractional calculus deals with derivatives or integrals for any type of order, whether real world numbers or complicated mathematical notions. It has recently gained popularity and favor. This is due to beneficial applications in science and engineering. Fractional calculus models can be used to study the effects of different parameters on electromagnetic blood flow, such as the strength and frequency of the applied electric and magnetic fields. For example, Sharma [13] used fractional calculus to model the flow of blood mixed with magnetic particles in a cylindrical tube. They found that the fractional parameter had a significant effect on the velocity of the blood and magnetic particles. The Caputo-Fabrizio fractional derivative is a new type of fractional derivative that does not have a singular kernel. It is a modified version of the Caputo fractional order derivative that is free from the singular kernel. Another derivative technique was introduced by Caputo & Fabrizio [14], which has been followed up in theoretical and applied studies of several real-world problems. It substitutes the singular kernel of Caputo derivative with an exponential function containing two forms for temporal variable and spatial variables respectively. Recent research has focused on using the Caputo-Fabrizio fractional derivative to model electromagnetic blood flow problems such as Morales-Delgado [15] studied the dynamics of oxygen diffusion through capillaries to tissues using Caputo-Fabrizio and Liouville-Caputo fractional derivatives. Furthermore, Ali et al. [16] considered an idealization of Walters-B fluid as a fractional model under study of MHD natural convection across a stationary horizontal plate. They study how some geometry specific magnetic field interact with the blood flow using Caputo-Fabrizio time fractional derivatives. Besides, Usman [17] used the Caputo-Fabrizio fractional derivative to model the flow of two-phase, unsteady magnetized blood with nano-particles in a cylindrical tube. They found that the Caputo-Fabrizio fractional derivative model was able to predict the velocity and temperature profiles more accurately than traditional models. Moreover, Riaz & Zafar [18] specifically focused on the application of the Caputo-Fabrizio fractional derivative in modeling blood flow through a circular tube under the influence of a magnetic field, highlighting its relevance in biomedical engineering and hemodynamics. Abdullah et al. [19] made use of the Caputo-Fabrizio fractional derivative for not only studying internal motion, but also electrically charged neuronal blood itself. Experiments notes that the velocities of blood and magnetic particles were all low in an external magnetic field. On the other hand when Reynolds number decreased both speeds increased.

The main objective of this research paper is to investigate the unsteady magnetic blood flow model in a stenosed artery, incorporating magnetic particles and considering the presence of electric and magnetic fields. The fluid was driven by pressure gradient and perpendicular magnetic field.

2. Mathematical model

Geometry of stenosis

The blood flow motion in this situation is axially symmetric and depends on the axial distance (z) and height of the stenosis, which grows with the vessel radius.

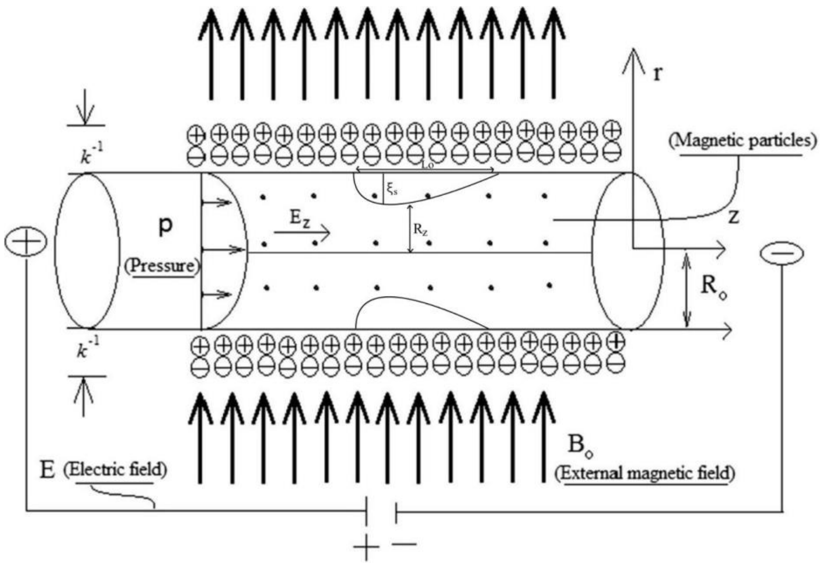


Figure 3.1: Electro-magneto blood flow in standardize stenosed artery model

Governing continuity and momentum equation

The governing continuity and momentum equations for the electromagnetic transport of blood mixed with magnetic particles in the cylindrical coordinates are provided by [8]:

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0 \tag{1}$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{\partial \theta} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial \rho}{\partial r} + \rho f_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right] \tag{2}$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial \rho}{\partial r} + \rho f_\theta + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right] \tag{3}$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial \rho}{\partial r} + \rho f_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] \tag{4}$$

Only one velocity component is non-zero, namely $u_z = u$ at z-direction. Therefore, u_r and $u_\theta = 0$. The axial component of velocity is independent of the angular location, hence $\frac{u_\theta}{\partial \theta} = 0$. The above governing equations from equation (1) - (4) are reduce to

$$\frac{\partial u_z}{\partial z} = 0 \tag{5}$$

$$\rho\left(\frac{\partial u_z}{\partial z}\right) = -\frac{\partial \rho}{\partial z} + \eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_z)}{\partial r}\right) + F_{em} + F_{uv} \tag{6}$$

Electromagnetic field and relative forces

The electromagnetic field force F_{em} and the force F_{uv} due to the motion between blood and magnetic particles can be represented as

$$\begin{aligned} F_{em} &= J \times B = \sigma(E + V \times B) \times B \\ &= -\sigma^2 B^2 u(r,t)\vec{j} - \rho_e E \\ &= -\sigma^2 B^2 u(r,t)\vec{j} - \rho_e E_z k \end{aligned} \tag{7}$$

$$F_{uv} = K_m N\{u(r,t) - v(r,t)\} \tag{8}$$

where E_z are the electric field in axial direction, $\vec{j}$ is the unit vector in the z-direction and $\rho_e = -\varepsilon\kappa^2\psi(r)$ is the net charge density. In $\kappa^2 = \frac{2z_0^2e_0^2n_0}{\varepsilon k_B T_\alpha}$, where ε is the dielectric constant, κ is the Debye-Hukel parameter, κ^{-1} is the thickness of electrical double layer, n_0 is the ionic concentration in the bulk phase, e_0 is the electronic charge, k_B is the Boltzmann constant, T_α is the absolute temperature K_m is the stokes constant and N is the number of magnetic particles per unit volume.

Bio-fluid velocity model

The dimensional equation with the presence of external pressure gradient and electromagnetic field:

$$\frac{\partial u(r,t)}{\partial t} = \frac{1}{\rho}(A_0 + A_1\cos(\omega t) + \nu\left(\frac{\partial^2 u(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,t)}{\partial r}\right) + \frac{K_m N\{v(r,t) - u(r,t)\}}{\rho} - \frac{\sigma^2 B^2 u(r,t)}{\rho} + \frac{\varepsilon\kappa^2\psi(r)E_z}{\rho}, A_0 > 0\,, \tag{9}$$

where A_0 and A_1 are the pulsatile components of the pressure gradient that cause systolic and diastolic pressure. The motion of magnetic particles in the blood flow is as follows:

$$m\frac{\partial v(r,t)}{\partial t} = K_m\{u(r,t) - v(r,t)\}\,, \tag{10}$$

where $u(r,t)$ and $v(r,t)$ represents the blood flow and magnetic particles velocity along the axis direction respectively and m is the magnetic particles average mass, ρ is the density of the fluid, and ν is the kinetic viscosity. The blood flow inside a circular cylinder with radius R_0 has the following initial and boundary conditions:

$$\begin{gathered} u(r,0) = 0, v(r,0) = 0, r \in [0, R_0]\,, \\ u(R_0,t) = 0, v(R_0,t) = 0, t > 0\,. \end{gathered} \tag{11}$$

Non-dimensional variables are introduced to analyze the non-dimensional model.

$$\begin{gathered} r^* = \frac{r}{R_0},\ \ t^* = \frac{t}{\lambda},\ \ u^* = \frac{u}{u_0},\ \ v^* = \frac{v}{v_0},\ \ \omega = \lambda\omega, \\ A_0^* = \frac{\lambda A_0}{\rho u_0},\ \ A_1^* = \frac{\lambda A_1}{\rho u_0},\ \ \psi^* = \frac{\psi}{\psi_0} \end{gathered} \tag{12}$$

The summation of the non-dimensional governing equation (after dropping dashes) can be obtained by inserting the non-dimensional parameter in (12) into each term of equation (9) to (11) :

$$D_t^{(\alpha)}u(r,t) = A_0 + A_1\cos(\omega t) + \frac{1}{Re}\left(\frac{\partial^2 u(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,t)}{\partial r}\right) + R\{v(r,t) - u(r,t)\} - Ha^2 u(r,t) + K^2\psi(r), A_0 > 0\,, \tag{13}$$

$$G\, D_t^{(\alpha)}v(r,t) = u(r,t) - v(r,t)\,, \tag{14}$$

$$\begin{gathered} u(r,0) = 0, v(r,0) = 0, r \in [0,1]\,, \\ u(1,t) = 0, v(1,t) = 0, t > 0\,. \end{gathered} \tag{15}$$

where $K^2 = \frac{\varepsilon\kappa^2 E_z\lambda}{\rho}$ is the electro-kinetic width, $Re = \frac{R_0^2}{\nu\lambda}$ is the Reynolds number, $R = \frac{K_m N\lambda}{\rho}$ is the particles concentration parameter, $Ha^2 = \frac{\sigma^2 B_0^2\lambda}{\rho}$ is the Hartmann number and $\mathrm{G} = \frac{\mathrm{m}}{\mathrm{K_m}\lambda}$ is the mass parameter of magnetic particles.

Caputo-Fabrizio fractional derivatives
Caputo and Fabrizio [14] renamed this NFDt after removing the singularity from the definition as follow

$$D_t^{(\alpha)} f(t) \frac{M(\alpha)}{(1-\alpha)} \int_a^t \dot{f}(\tau) exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right) d\tau\,, \tag{16}$$

with $\alpha \in [0,1]$ and $a \in [-\infty, t)$, $f \in H^1(a,b)$, $b > a$ where H^1 is the class of all integrable functions on *[a,b].* M(α) is a normalization function with the property that M(0) = M(1) = 1. When *f(t)* is constant, Caputo-Fabrizio derivative of a function equals zero. It is worth mentioning that the Caputo-Fabrizio fractional-order model is an extension of the integer order model. Moreover, NFDt, the new definition of fractional derivative, loses not just singularity but also another property. It occurs as the old definition UFDt at zero for a constant f(t). Noteworthily, $f(t) \notin H^1(a,b)$ can also be operated by NFDt. The equivalent form of the definition in Eq. (16) is as follows:

$$D_t^{(\alpha)} f(t) = \frac{N(\alpha)}{\alpha} \int_a^t \dot{f}(\tau) exp\left(-\frac{(t-\tau)}{\sigma}\right) d\tau \tag{17}$$

Solution to the problem
To simplify, after applying Laplace transform to the equations (13) and (14) along with the boundary conditions (15), the fluid flow model can be expressed like this:

$$\frac{s\bar{u}(r,s)}{s+\alpha(1-s)} = \frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2} + \frac{1}{Re}\left(\frac{\partial^2 \bar{u}(r,s)}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}(r,s)}{\partial r}\right) + R\{\bar{v}(r,s) - \bar{u}(r,s)\} - Ha^2 \bar{u}(r,s) + \frac{K^2 \psi(r)}{s}, \tag{18}$$

$$G \cdot \frac{s\bar{v}(r,s)}{s+\alpha(1-s)} = \bar{u}(r,s) - \bar{v}(r,s)\,, \tag{19}$$

$$\bar{u}(1,s) = 0, \bar{v}(1,s) = 0\,. \tag{20}$$

Using a finite Hankel transform with respect to the radial coordinate,r and considering the given boundary conditions in (20) leads to:

$$\bar{u}_H(\varsigma_n, s)\left(\frac{s}{s+\alpha(1-s)} - R\left(\frac{s+\alpha(1-s)}{s+sG+\alpha(1-s)}\right) + Ha^2 + R + \frac{\varsigma_n^2}{Re}\right) = \left(\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2}\right)\frac{J_1(\varsigma_n)}{\varsigma_n} + \frac{K^2}{s}\frac{\varsigma_n}{\varsigma_n^2+K^2}J_1(\varsigma_n)\,. \tag{21}$$

The expression, $\bar{u}_H(\varsigma_n, s) = \int_0^1 r\bar{u}(r,s)\, J_0(\varsigma_n, s) dr$ represents the finite Hankel transform of the velocity function $\bar{u}(r,s)$, where $\bar{u}(r,s) = \mathcal{L}[u(r,t)]$ and $\varsigma_n, n = 1,2,\ldots$ are the positive roots of the equation $J_0(x) = 0$. In this context, J_0 denotes the Bessel function of order zero of the first kind. By simplifying the coefficient of $\bar{u}_H(\varsigma_n, s)$ in equation (21), we obtain

$$\bar{u}_H(\varsigma_n, s) = \frac{s^2 m_{5n} + s m_{6n} + \alpha^2}{s^2 m_{2n} + s m_{3n} + m_{4n}}\left[\left(\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2}\right)\frac{J_1(\varsigma_n)}{\varsigma_n} + \frac{K^2}{s}\frac{\varsigma_n}{\varsigma_n^2+K^2}J_1(\varsigma_n)\right], \tag{22}$$

$$\bar{u}_H(\varsigma_n, s) = \left(\frac{m_{9n}}{s-m_{7n}} + \frac{m_{10n}}{s-m_{8n}}\right)\left[\left(\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2}\right)\frac{J_1(\varsigma_n)}{\varsigma_n} + \frac{K^2}{s}\frac{\varsigma_n}{\varsigma_n^2+K^2}J_1(\varsigma_n)\right]. \tag{23}$$

Here are the parameters outlined in equations (24) which are aimed at simplifying the coefficient of $\bar{u}_H(\varsigma_n, s)$ as presented in equation (21):

$$m_{1n} = Ha^2 + R + \frac{\varsigma_n}{Re},$$
$$m_{2n} = 1 + G - \alpha - R - R\alpha^2 + 2R\alpha + m_{1n} + \alpha^2 m_{1n} - 2\alpha m_{1n} - G m_{1n} - G\alpha m_{1n},$$
$$m_{3n} = \alpha + 2R\alpha^2 - 2R\alpha - 2m_{1n}\alpha^2 + 2\alpha m_{1n} + G\alpha m_{1n},$$
$$m_{4n} = \alpha^2 m_{1n} - R\alpha^2,$$
$$m_{5n} = 1 + \alpha^2 - 2\alpha + G - G\alpha,$$
$$m_{6n} = -2\alpha^2 + 2\alpha + G\alpha,$$
$$m_{7n} = \frac{-m_{3n} + \sqrt{m_{3n}^2 - 4m_{2n}m_{4n}}}{2m_{2n}},$$
$$m_{8n} = \frac{-m_{3n} - \sqrt{m_{3n}^2 - 4m_{2n}m_{4n}}}{2m_{2n}},$$
$$m_{9n} = \frac{m_{7n}^2 m_{5n} + m_{7n}m_{6n} + \alpha^2}{m_{7n} - m_{8n}},$$
$$m_{10n} = \frac{m_{8n}^2 m_{5n} + m_{8n}m_{6n} + \alpha^2}{m_{8n} - m_{7n}}, \tag{24}$$

Laplace transform of the image function $\bar{u}_H(\varsigma_n, s)$ discussed in Eq. (23) is obtained by using the Robotnov and Hartley's functions

$$\mathcal{L}^{-1}\left[\frac{1}{s^a + b}\right] = F_a(-b,t) = \sum_{n=0}^{\infty} \frac{(-b)^n\, t^{(n+1)a-1}}{\Gamma\left((n+1)a\right)}, a > 0 \tag{25}$$

$$\mathcal{L}^{-1}\left[\frac{s^c}{s^a + b}\right] = R_{a,c}(-b,t) = \sum_{n=0}^{\infty} \frac{(-b)^n\, t^{(n+1)a-1-c}}{\Gamma\left((n+1)a-c\right)}, \mathrm{Real}(a-c) > 0 \tag{26}$$

$$u_H(\varsigma_n, t) = \frac{J_1(\varsigma_n)}{\varsigma_n}\left[(e^{m_{7n}t} - 1)\left(\frac{\varsigma^2 K^2 m_{9n}}{m_{7n}(\varsigma^2 + K^2)} + \frac{A_0 m_{9n}}{m_{7n}}\right) + m_{9n}A_1 e^{m_{7n}t} * \cos\omega t + m_{10n}A_1 e^{m_{8n}t} * \cos\omega t + (e^{m_{8n}t} - 1)\left(\frac{\varsigma^2 K^2 m_{10n}}{m_{8n}(\varsigma^2 + K^2)} + \frac{A_0 m_{10n}}{m_{8n}}\right)\right]. \tag{27}$$

Equation (27) denotes the convolution product of f and g, denoted as $f * g$, which the product can be calculated as follows:

$$(f * g)(t) = \int_0^t f(\tau)(t - \tau)d\tau\,. \tag{28}$$

Fluid velocity

By using the inverse Hankel transform of equation (27) one can obtain the bio-fluid flow.

$$u(r,t) = 2\sum_{n=1}^{\infty} \frac{J_0(r\varsigma_n)}{J_1^2(\varsigma_n)} \times \bar{u}_H(\varsigma_n, t), \tag{29}$$

$$u(r,t) = 2\sum_{n=1}^{\infty} \frac{J_0(r\varsigma_n)}{\varsigma_n J_1(\varsigma_n)}\left[(e^{m_{7n}t} - 1)\left(\frac{\varsigma^2 K^2 m_{9n}}{m_{7n}(\varsigma^2 + K^2)} + \frac{A_0 m_{9n}}{m_{7n}}\right) + m_{9n}A_1 e^{m_{7n}t} * \cos\omega t + m_{10n}A_1 e^{m_{8n}t} * \cos\omega t + (e^{m_{8n}t} - 1)\left(\frac{\varsigma^2 K^2 m_{10n}}{m_{8n}(\varsigma^2 + K^2)} + \frac{A_0 m_{10n}}{m_{8n}}\right)\right], \tag{30}$$

Magnetic particles velocity

The velocity of the magnetic particles mixed with blood is obtained from equation (19)

$$v(r,t) = m_{12n}(1 - m_{11n})[u(r,t) * e^{-m_{12n}t}], \text{ for } 0 < \alpha \le 1\,. \tag{31}$$

The parameters presented in equation (31) are as follows:

$$m_{11n} = \frac{1-\alpha}{G-\alpha+1}, \qquad m_{12n} = \frac{\alpha}{G-\alpha+1}. \tag{32}$$

3. Numerical results and discussion

Our goal was to obtain the blood flow stream parameters from the new definition of Caputo–Fabrizio fractional derivative solved by using both Laplace and finite Hankel transforms. The simulation using Mathcad used the following parameters:

$$A_0 = 0.5,\ A_1 = 0.6,\ G = 0.5,\ R = 0.5,\ Re = 5,\ \omega = \frac{\pi}{4},\ t = 0.2,\ K = 1 \text{ and } Ha = 2.$$

Velocity profiles

Figure 1 depicts the effect of fractional order derivatives ($\alpha = 0.4, 0.6, 0.8, 1.0$) on velocities. The fractional parameter, enhanced both blood flow and magnetic particle velocities. Figure 1 illustrates how magnetic fields affect blood flow and magnetic particles. Magnetic particle velocities appear to decrease with increasing Hartmann number. A magnetic field in the blood flow tends to slow down the particle motion due to the rise in resistive force.

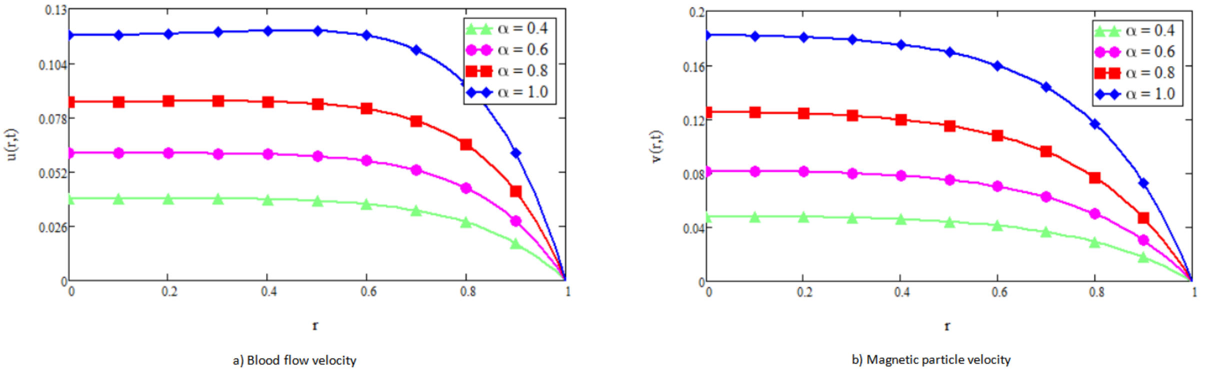


Figure 1: Profile of axial velocities of *u(r, t)* and *v(r, t)* for
$A_0 = 0.5,\ A_1 = 0.6,\ G = 0.5,\ R = 0.5,\ Re = 5,\ \omega = \pi/4,\ Ha = 2,\ t = 0.2,\ K = 1$
and $\alpha = 0.4,\ 0.6,\ 0.8,\ 1$ against r

Figure 2 shows significant decreases in both velocities as *Ha* increases. High values of *Ha* increased the resistive effect on blood flow, resulting in a significant decrease in magnetic particle velocity.

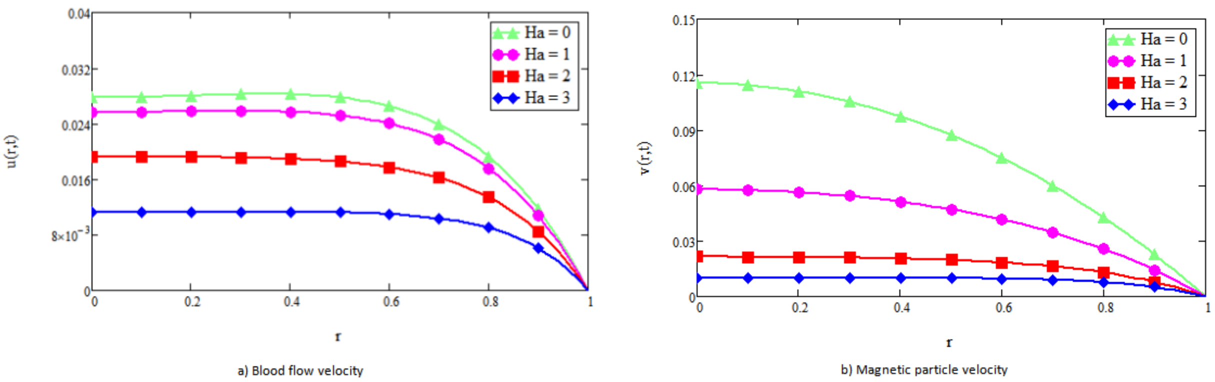
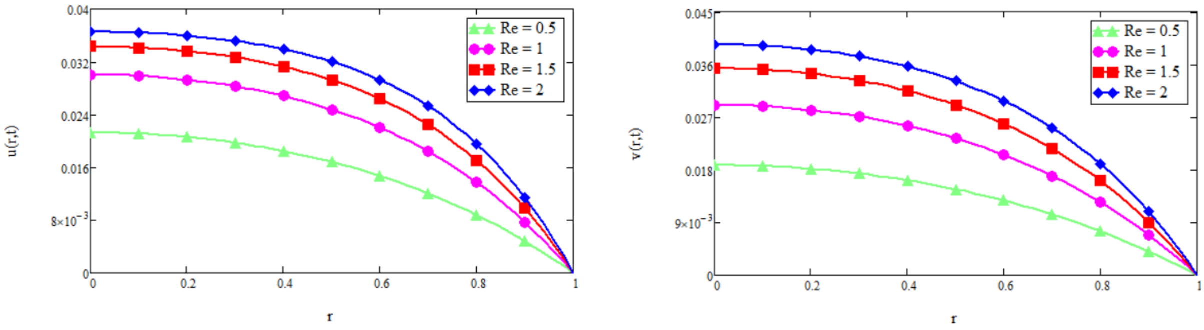


Figure 2: Profile of axial velocities of *u(r, t)* and *v(r, t)* for
$A_0 = 0.5,\ A_1 = 0.6,\ G = 0.5,\ R = 0.5,\ Re = 5,\ \omega = \pi/4,\ \alpha = 0.4,\ t = 0.2,\ K = 1,$
and $Ha = 0,\ 1,\ 2,\ 3$ against r

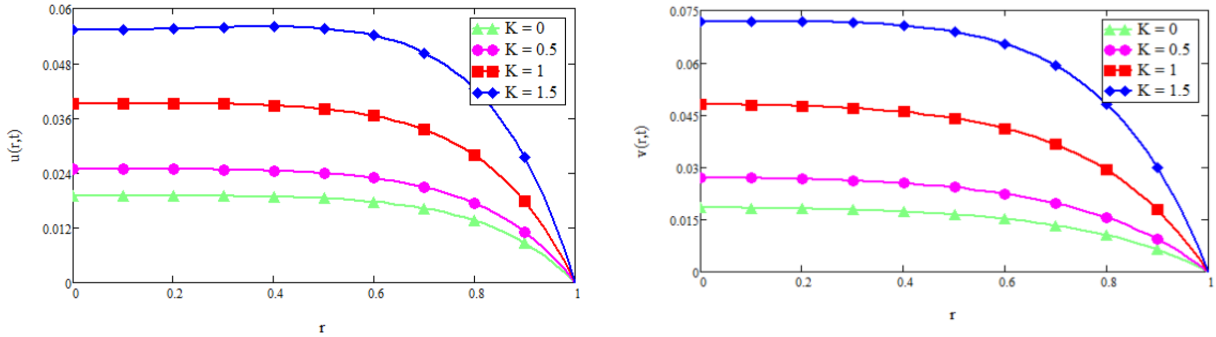
Figure 3 illustrates the consequences of Reynolds number, Re increased with relative to velocities, indicating a decrease in blood viscosity and improved flow



a) Blood flow velocity b) Magnetic particle velocity

Figure 3: Profile of axial velocities of *u(r, t)* and *v(r, t)* for
$A_0 = 0.5$, $A_1 = 0.6$, $G = 0.5$, $R = 0.5$, $Ha = 2$, $\omega = \pi/4$, $\alpha = 0.4$, $t = 0.2$, $K = 1$
and $Re = 0.5, 1, 1.5, 2$ against r.

Figure 4 shows the external electric field effects on blood flow and magnetic particle velocities. At zero electric field, the velocity of blood flow is greater than magnetic particles distributions due to drag force. Increasing the external electric field accelerates the velocity of magnetic particles due to collisions with charged particles.



a) Blood flow velocity b) Magnetic particle velocity

Figure 4: Profile of axial velocities of *u(r, t)* and *v(r, t)* for
$A_0 = 0.5$, $A_1 = 0.6$, $G = 0.5$, $R = 0.5$, $Re = 5$, $\omega = \pi/4$, $\alpha = 0.4$, $t = 0.2$, $Ha = 2$,
and $K = 0, 0.5, 1, 1.5$ against r.

4. Conclusion

The blood flow mixed with magnetic particles was subjected to the external electro-kinetic energy and magnetic particles. The flow was modeled by using Caputo-Fabrizio fractional order derivative with singular kernel.The exact analytical solutions for the blood flow and magnetic particles velocities were obtained by means of Laplace and finite Hankel transform. Graphical plots were generated using the Mathcad software. The fractional parameter increases both blood flow and magnetic particle velocities, while increasing the Hartmann number decreases them because of the rising resistive force. Higher Reynolds numbers improve flow and decrease blood viscosity, while high Hartmann numbers significantly slow down magnetic particles. At zero electric field, drag force causes the velocity of blood flow to exceed the velocity of magnetic particles.

Acknowledgement
This researcher would like thank the people who have supported the development of ideas and knowledge. This thesis would not have been the same if it was not for their continual support and attention.

References
[1] C. Sanyal, K. Das and S. Debnath (2007), “Effect of Magnetic Field on Pulsatile Blood Flow through an Inclined Circular Tube with Periodic Body Acceleration,” Journal of Physical Science, Vol. 11,, pp. 43-56.
[2] Mohan, V., Prasad, V., Varshney, N.K. and Gupta, P.K. (2013) Effect of Magnetic Field on Blood Flow (Elastico-Viscous) Under Periodic Body Acceleration in Porous Medium. IOSR Journal of Mathematics, 6, 43-48.

[3] Shah, S. and Kumar, R. (2020). Mathematical modeling of blood flow with the suspension of nanoparticles through a tapered artery with a blood clot. Frontiers in Nanotechnology.

[4] Ponalagusamy, R. and Manchi, R. (2021). Mathematical study on two-fluid model for flow of k–l fluid in a stenosed artery with porous wall. SN Applied Sciences.

[5] Hamza, Z. A., Al-Azawy, M. G., Alkinani, A. A., Al-Waaly, A. A., & Ahmed, B. (2022). Evaluate the hemodynamic factors' effects on stenosed arteries with the carreau model. World Journal of Advanced Research and Reviews, 16(1), 622-631.

[6] Shaikh, F., Hınçal, E., & Shaikh, A. A. (2023). Comparative analysis of numerical simulations of blood flow through the segment of an artery in the presence of stenosis. Journal of Applied Mathematics and Computational Mechanics, 22(2), 49-61.

[7] Sharma S, Singh U, Katiyar V (2015) Magnetic field effect on flow parameters of blood along with magnetic particles in a cylindrical tube. J Magn Magn Mater 377:395–401

[8] Shah NA, Vieru D, Fetecau C (2016a) Effects of the fractional order and magnetic field on the blood flow in cylindrical domains. J Magn Magn Mater 409:10–19

[9] Omamoke, O., & Adeleke, A. (2020). Consequences of Thermal Radiation and Heat Source in Combination with Magnetohydrodynamic Blood Flow. International Journal of Heat and Mass Transfer, 150, 119252.

[10] Yang, L., & Wang, H. (2021). A Multi-Electrode Electromagnetic Flow Detection Method for Non-Invasive Blood Vessel Measurement. Journal of Medical Engineering & Technology, 45(6), 289-2.

[11] Selvi, R., Ponalagusamy, R., & Padma, R. (2021). Influence of electromagnetic field and thermal radiation on pulsatile blood flow with nanoparticles in constricted porous artery. International Journal of Applied and Computational Mathematics, 7(6).

[12] Caputo, M. (2008). Fractional calculas and applied analysis. An International Journal for Theory and Applications. 11(1): 73-85.

[13] Sharma S, Singh U, Katiyar V (2015) Magnetic field effect on flow parameters of blood along with magnetic particles in a cylindrical tube. J Magn Magn Mater 377:395–401.

[14] Caputo, M. and Fabrizio, M. (2015) A New Definition of Fractional Derivative without Singular Kernel. Progress in Fractional Differentiation and Applications, 2, 73-85.

[15] Morales-Delgado VF, Gmez-Aguilar JF, Saad KM, Khan MA, Agarwa P (2019) Analytic solution for oxygen diffusion from capillary to tissues involving external force effects: a fractional calculus approach. Phys A 523:48–65.

[16] Shah N A, Vieru D and Fetecau C (2016) Journal of Magnetism and Magnetic Materials. 409, 10-19.

[17] Usman M, Hamid M, Khan U, Tauseef S, Din M, Asad M, Wang W (2018) Differential transform method for unsteady nanofluid flow and heat transfer. Alex Eng J 50(3):1867–1875.

[18] Riaz, M. B. and Zafar, A. A. (2018). Exact solutions for the blood flow through a circular tube under the influence of a magnetic field using fractional caputo- fabrizio derivatives. Mathematical Modelling of Natural Phenomena, 13(1), 8.

[19] Abdullah, M., Butt, A., R., Raza, N., Alshomrani, A. S. & Alzahrani, A. K. (2018). Analysis of blood flow with nanoparticles induced by uniform magnetic field through a circular cylinder with fractional Caputo derivatives. Journal of Magnetism & Magnetic Materials. 446: 28-36.