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Sombor Index, Reduced Sombor Index, and Average Sombor Index of Coprime Graph Associated to the Dihedral Groups of Order $2n$

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Abstract
Some of the main applications of graphs in representing chemical structures involve the use of graph theory and topological indices. Various types of topological indices have been introduced in mathematics. Recently, new knot degree-based molecular structures were proposed, namely the Sombor index, reduced Sombor index, and average Sombor index. One of the studies on topological indices by Yatin et al. explains the calculation of topological indices from coprime graphs for dihedral groups on the Hyper-Wiener index and the Padmakar-Ivan index. The coprime graph of a group G, denoted by Γ_G, is a graph whose vertices are elements of G, and two distinct vertices u and v are adjacent if and only if $(|u|, |v|) = 1$. The results of this research are the general formulas for the Sombor index, reduced Sombor index, and average Sombor index on coprime graphs of the dihedral group D_{2n}.
Keywords: Coprime graph, Dihedral group, Sombor index, Reduced sombor index, Average sombor index.

Introduction

Graph theory was born in 1736 through Euler's writings, which included efforts to solve the Königsberg bridge problem, a famous puzzle in Europe [1]. In 1847, G.R. Kirchhoff (1824-1887) succeeded in developing the theory of trees, which was used to solve electrical network problems. Ten years later, Cayley (1821-1895) applied the concept of trees to explain chemical problems, specifically hydrocarbons [1]. Problems in chemistry are also related to graphs, which are widely used to represent molecular structures and chemical compounds. Research studies on graphs are very interesting, particularly those involving coprime graphs and non-coprime graphs [2].

Coprime graphs are graphs that have unique properties associated with prime numbers, while non-coprime graphs present an interesting contrast to explore [3]. Several studies focus on coprime graphs, including by Nurhabibah et al, in 2022 [4]. The results obtained from this study show that the harary index of coprime graphs for groups of modulo integers, whose order is a power of prime numbers, is the square of the number of members of the group minus one [4]. Besides the coprime graphs of the modulo integers, there are also coprime graphs of the dihedral group. The coprime graph of a dihedral group, denoted by $\Gamma_{D_{2n}}$, is a graph representation of a dihedral group D_{2n}, where the elements of the dihedral group are vertices. Alkadar et al. in 2021 explain the characteristics of coprime graphs of subgroups in symmetry groups. Their research shows that the product of each subgroup of order two different prime numbers forms a complete 3-partite graph, and each subgroup with elements of the same order or multiples forms a star graph.

Some of the main applications of graphs in the representation of chemical structures involve using graph theory and topological indices [5]. The topological index of a graph is a number that remains invariant under graph automorphisms. Various types of topological indices have been introduced in mathematics, such as the Wiener index, Szeged index, Padmakar-Ivan index, and Randić index [6]. Recently, a new knot degree-based molecular structure called the Sombor index.The degree $d_u$of vertex $u \in V(G)$ is the number of vertices adjacent to u. If there is an edge from vertex u to vertex v show this with uv (or vu) [7]. Since G is a simple graph, we have $u \neq v$ [8] . This topological index is motivated by the geometric interpretation of the degree radius of an edge, which is the distance from the origin to the ordered pair (d_u, d_v), where $d_u \leq d_v$[9]. We also have its generalization, called the reduced Sombor index. Besides that, we have another generalization called the average Sombor index.

Previous studies provide the calculation of topological indices from coprime graphs for dihedral groups on the Hyper-Wiener index and the Padmakar-Ivan index but have not yet provided the Sombor index and its generalizations [10]. Based on several previous studies on coprime graphs, it is interesting to study other topological indices of these graphs, namely the sombor index, reduced sombor index, and average Sombor index. In this research, the coprime graph observed is the coprime graph of the dihedral group D_{2n} of odd prime power order.

Materials and methods

The research procedures for completing this study are:

1. Exploring relevant sources to aid in understanding related research problems. This involves delving into the literature on graph theory, groups, and Sombor indices.
2. Analyzing and observing graph patterns to identify the Sombor index, reduced Sombor index, and average Sombor index formed in coprime graphs over dihedral groups D_{2n}. These patterns are obtained through research conducted on examples relevant to the investigation.
3. Formulating temporary hypotheses regarding the characteristics of the Sombor index, reduced Sombor index, and average Sombor index in coprime graphs over dihedral groups D_{2n}. These hypotheses are then proven through mathematical proof to ascertain the actual characteristics of these indices. If the hypotheses are proven, they are transcribed into theorems concerning the Sombor index, reduced Sombor index, and average Sombor index on coprime graphs of dihedral groups D_{2n}.
4. Conclude from the results obtained from the research conducted.

Results and discussion

Definition 1 [12] Let G be a group. The group G is said to be the dihedral group of order $2n$, $n \geq 3$, is a group constructed by two elements a, b with:

$$G = D_{2n} = \langle a , b \mid a^n = e, b^2 = e, bab^{-1} = a^{-1} \rangle.$$

The element b is an element of order two, and is called a reflection element, and an element a, which has an order of more than $n \geq 3$, is said to be a rotation element. The order of the dihedral group D_{2n} is $2n$. For example, for $n = 3$, we obtain the dihedral group $D_6 = \{e, a, a^2, b, ab, a^2b\}$.

Definition 2 [13] The coprime graph of a group G, denoted by Γ_G, is a graph whose vertices are elements of G and two distinct vertices u and v are adjacent if and only if $(|a|, |b|) = 1$.

Theorem 1 [13] Suppose $n = p^k$, $p \neq 2$ and p is a prime number, for every $k \in \mathbb{N}$, then the coprime graph $\Gamma_{D_{2n}}$is a complete tripartite graph.

Definition 3 [14] Suppose we are given a graph G with a set of vertices $V(G)$ and a set of edges $E(G)$. Then the sombor index of G, which is denoted by $SO(G)$ is:

$$SO(G) = \sum_{u,v \in E(G)} \sqrt{{d_u}^2 + {d_v}^2}.$$

Definition 4 [8] Suppose we are given a graph G with a set of vertices $V(G)$ and a set of edges $E(G)$. Then, the reduced Sombor index of G, which is denoted by $SO_{red}(G)$ is:

$$SO_{red}(\,G) = \sum_{u,v\,\in E(G)} \sqrt{(d_u - 1)^2 + (d_v - 1)^2}.$$

Definition 5 [8] Suppose we are given a graph G with a set of vertices V(G) and a set of edges E(G). Then the avearage sombor index of G, which is denoted by $SO_{avr}(G)$ is:

$$SO_{avr}(\,G) = \sum_{u,v\,\in E(G)} \sqrt{\left(d_u - \frac{2m}{n}\right)^2 + \left(d_v - \frac{2m}{n}\right)^2}$$

with $m = |E(G)|$ and $n = |V(G)|$.
For example, for the dihedral group $D_6 = \{e, a, a^2, b. ab, a^2b\}$, the order of D_6 are as follows:

Table 1. Element and Order D_6

Element	e	a	a^2	b	ab	a^2b
Order	1	3	3	2	2	2

After determining the order of the dihedral group elements, we can create a coprime graph of the elements in D_6, accordance with Definition 2.

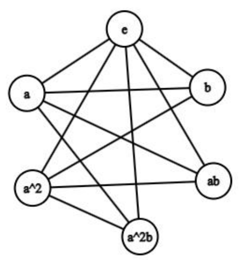

Figure 1. The coprime graph Γ_{D_6}

A coprime graph $\Gamma_{D_6}$is a graph as in Theorem 1 with $p = 3$ and $k = 1$, formed a complete tripartite graph. The degrees of each vertex are: $deg\,(e) = 5, deg\,(a) = 4, deg\,(\,a^2) = 4, deg\,(\,b) = 3, deg\,(ab) = 3, deg\,(a^2b) = 3$. It can be seen that the degrees of the rotation elements are four, and the reflection elements are three. The coprime graph formed is a complete tripartite graph. So to get the Sombor index we can divide it into three cases.

- Case 1
  In the first case, the relationship between the identity element and the rotation element is calculated first.

$$SO\left(\Gamma_{D_6}\right) = \textstyle\sum_{u,v\,\in E(\Gamma_{D_6})} \sqrt{{d_u}^2 + {d_v}^2} = \sqrt{{d_e}^2 + {d_a}^2} + \sqrt{{d_e}^2 + {d_{a^2}}^2}$$

$$= \sqrt{5^2 + 4^2} + \sqrt{5^2 + 4^2} = 2\sqrt{41}.$$

- Case 2
  In the second case, the relationship between the identity element and the reflection elements is calculated.

$$SO\left(\Gamma_{D_6}\right) = \sum_{u,v\,\in E(\Gamma_{D_6})} \sqrt{{d_u}^2 + {d_v}^2}$$

$$= \sqrt{{d_e}^2 + {d_b}^2} + \sqrt{{d_e}^2 + {d_{ab}}^2} + \sqrt{{d_e}^2 + {d_{a^2b}}^2}$$

$$= \sqrt{5^2+3^2} + \sqrt{5^2+3^2} + \sqrt{5^2+3^2}$$

$$= 3\sqrt{34}.$$

- Case 3
  And finally, the relationship between the rotation element and the reflection element is calculated.

$$SO\left(\Gamma_{D_6}\right) = \sum_{u,v\,\in E(\Gamma_{D_6})} \sqrt{{d_u}^2 + {d_v}^2}$$

$$= \sqrt{{d_a}^2 + {d_b}^2} + \sqrt{{d_a}^2 + {d_{ab}}^2} + \sqrt{{d_a}^2 + {d_{a^2b}}^2} + \sqrt{{d_{a^2}}^2 + {d_b}^2}$$
$$+ \sqrt{{d_{a^2}}^2 + {d_{ab}}^2} + \sqrt{{d_{a^2}}^2 + {d_{a^2b}}^2}$$

$$= \sqrt{4^2+3^2} + \sqrt{4^2+3^2} + \sqrt{4^2+3^2} + \sqrt{4^2+3^2} + \sqrt{4^2+3^2}$$
$$+ \sqrt{4^2+3^2}$$

$$= 6\sqrt{25} = 30.$$

Based on the three cases above, in general, the Sombor index, the reduced Sombor index, and the average Sombor index of a coprime graph Γ_{D_6} is:

$$SO\left(\Gamma_{D_6}\right) = 2\sqrt{41} + 3\sqrt{34} + 30.$$

From the example above, we generalize it to a more formula. Let the rotation elements, $P_1 = \{a, a^2, a^3, \dots, a^{n-1}\}$, and the reflection elements $P_2 = \{b, ab, a^2b, \dots, a^{n-1}b\}$. Based on the study previously conducted [11], the degree of each vertex is :

1. $deg\,(e) = 2n-1$,
2. $deg\,(x) = n+1$, for all $x \in P_1$
3. $deg\,(y) = n$, for all $y \in P_2$

Thus we have the following theorem.

Theorem 2 Let D_{2n} be a dihedral group with $n = p^k$, p is odd prime number, and $k \in \mathbb{N}$. Then, the Sombor index of the coprime graph is given by the following formula:

$$SO\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{(2n-1)^2 + (n-1)^2} + (n)\sqrt{(2n-1)^2 + (n)^2} +$$
$$n(n-1)\sqrt{(n+1)^2 + (n)^2}.$$

Proof : Since the group is complete bipartite, we partitioned the group into three partitions: $\{e\}$, P_1, and P_2. The set P_1 consist of all the rotation elements and P_2 consist of all the reflection elements. We have that $\deg\,(e) = 2n-1$, $\deg(x) = n+1$ for all $x \in P_1$ and $\deg(y) = n$ for all $y \in P_2$. We divide the cases into three cases to get the Sombor index:

- In the first case, the relationship between the identity element and the rotation element is calculated first.

$$SO\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_1 \in E\,\Gamma_{(D_{2n})}} \sqrt{{d_e}^2 + {d_{P_1}}^2}$$
$$= \sum_{e,P_1 \in E\,\Gamma_{(D_{2n})}} \sqrt{(2n-1)^2 + (n+1)^2}$$

Since *e* is adjacent to every vertices of P_1, and $|P_1| = n-1$, then

$$SO\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_1 \in E\,\Gamma_{(D_{2n})}} \sqrt{(2n-1)^2 + (n+1)^2}$$
$$= (n-1)\sqrt{(2n-1)^2 + (n+1)^2}$$

- In the second case, the relationship between the identity element and the reflection elements is calculated

$$SO\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_2 \in E\,\Gamma_{(D_{2n})}} \sqrt{{d_e}^2 + {d_{P_2}}^2}$$
$$= \sum_{e,P_2 \in E\,\Gamma_{(D_{2n})}} \sqrt{(2n-1)^2 + n^2}$$

Since e is adjacent to every vertices of P_2, and $|P_2| = n$, then

$$SO\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_2 \in E\,\Gamma_{(D_{2n})}} \sqrt{(2n-1)^2 + n^2} = n\sqrt{(2n-1)^2 + n^2}$$

- And finally, the relationship between the rotation element and the reflection element is calculated

$$SO\left(\Gamma_{D_{2n}}\right) = \sum_{\substack{u \in P_1 \\ v \in P_2}} \sqrt{{d_u}^2 + {d_v}^2} = \sum_{\substack{,u \in P_1 \\ v \in P_2}} \sqrt{(n+1)^2 + n^2}.$$

Since $|P_1| = n-1$ and $|P_2| = n$, then

$$SO\left(\Gamma_{D_{2n}}\right) = \sum_{\substack{,u \in P_1 \\ v \in P_2}} \sqrt{(n+1)^2 + n^2} = n(n-1)\sqrt{(n+1)^2 + n^2}.$$

Based on the three cases above, in general, the Sombor index of a coprime graph $\Gamma_{D_{2n}}$ is:

$$SO\,(G) = (n-1)\sqrt{(2n-1)^2 + (n-1)^2} + (n)\sqrt{(2n-1)^2 + (n)^2} + n(n-1)\sqrt{(n+1)^2 + (n)^2}. \blacksquare$$

Similarly, we also managed to find the general formula for the reduced sombor index.

Theorem 3 Let D_{2n} be a dihedral group with $n = p^k$, p is an odd prime number, and $k \in \mathbb{N}$. Then the reduced sombor index of the coprime graph is given by the following formula:

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{(2n-2)^2 + n^2} + (n)\sqrt{(2n-2)^2 + (n-1)^2}$$
$$+\; n(n-1)\sqrt{n^2 + (n+1)^2}.$$

Proof : Since the group is complete bipartite, we partitioned the group into three partitions: $\{e\}$, P_1, and P_2. The set P_1 consist of all the rotation elements and P_2 consist of all the reflection elements. We have that $\deg(e) = 2n-1$, $\deg(x) = n+1$ for all $x \in P_1$ and $\deg(y) = n$ for all $y \in P_2$.
We divide the cases into three cases to get the reduced sombor index:

- In the first case, the relationship between the identity element and the rotation element is calculated first.

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_1 \in E\,\Gamma_{(D_{2n})}} \sqrt{(d_e - 1)^2 + (d_{P_1} - 1)^2}$$
$$= \sum_{e,P_1 \in E\,\Gamma_{(D_{2n})}} \sqrt{(2n-1-1)^2 + (n+1-1)^2}$$
$$= \sum_{e,P_1 \in E\,\Gamma_{(D_{2n})}} \sqrt{(2n-2)^2 + n^2}.$$

Since *e* is adjacent to every vertices of P_1, and $|P_1| = n-1$, then

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_1 \in E\,\Gamma(D_{2n})} \sqrt{(2n-2)^2+n^2}$$
$$= (n-1)\sqrt{(2n-2)^2+n^2}.$$

- In the second case, the relationship between the identity element and the reflection elements is calculated

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_2 \in E\,\Gamma(D_{2n})} \sqrt{(d_e-1)^2+(d_{P_2}-1)^2}$$
$$= \sum_{e,P_2 \in E\,\Gamma(D_{2n})} \sqrt{(2n-2)^2+(n-1)^2}.$$

Since *e* is adjacent to every vertices of P_2, and $|P_2| = n$, then

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_2 \in E\,\Gamma(D_{2n})} \sqrt{(2n-2)^2+(n-1)^2}$$
$$= n\sqrt{(2n-2)^2+(n-1)^2}.$$

- And finally, the relationship between the rotation element and the reflection element is calculated

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = \sum_{\substack{,u\in P_1\\ v\in P_2}} \sqrt{(d_u-1)^2+(d_v-1)^2}$$
$$= \sum_{\substack{,u\in P_1\\ v\in P_2}} \sqrt{(n+1-1)^2+(n-1)^2}$$
$$= \sum_{\substack{,u\in P_1\\ v\in P_2}} \sqrt{(n)^2+(n-1)^2}.$$

Since P_1 is adjacent to every vertices of P_2, $|P_1| = n-1$ and $|P_2| = n$, then

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = \sum_{\substack{,u\in P_1\\ v\in P_2}} \sqrt{(n)^2+(n-1)^2}$$
$$= n(n-1)\sqrt{n^2+(n-1)^2}.$$

Based on threee cases above In general, the reduced Sombor index of a coprime graph $\Gamma_{D_{2n}}$ is:

$$SO_{red}\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{(2n-2)^2+n^2} + n\sqrt{(2n-2)^2+(n-1)^2}$$
$$+n(n-1)\sqrt{n^2+(n-1)^2}. \blacksquare$$

Using a similar approach, we have also successfully derived the general formula for the average Sombor index

Theorem 4 Let D_{2n} be a dihedral group with $n = p^k$, p is an odd prime number, and $k \in \mathbb{N}$. Then the reduced Sombor index of the coprime graph is given by the following formula:

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{\left(2n-1-\frac{2m}{n}\right)^2+\left(\mathrm{n}+1-\frac{2m}{n}\right)^2} + n\sqrt{\left(2n-1-\frac{2m}{n}\right)^2+(n-\frac{2m}{n})^2}$$

$$+n(n-1)\sqrt{\left(n+1-\frac{2m}{n}\right)^2+\left(n-\frac{2m}{n}\right)^2}.$$

Proof : Since the group is complete bipartite, we partitioned the group into three partitions: $\{e\}$, P_1, and P_2. The set P_1 consist of all the rotation elements and P_2 consist of all the reflection elements. We have that $\deg(e) = 2n-1$, $\deg(x) = n+1$ for all $x \in P_1$ and $\deg(y) = n$ for all $y \in P_2$. We divide the cases into three cases to get the average sombor index:

- In the first case, the relationship between the identity element and the rotation element is calculated first.

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_1 \in E\,\Gamma(D_{2n})} \sqrt{\left(d_e-\frac{2m}{n}\right)^2+\left(d_{P_1}-\frac{2m}{n}\right)^2}$$
$$= \sum_{e,P_1 \in E\,\Gamma(D_{2n})} \sqrt{(2n-1-\frac{2m}{n})^2+(n+1-\frac{2m}{n})^2}.$$

Since *e* is adjacent to every vertices of P_1, and $|P_1| = n-1$, then

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_1 \in E\,\Gamma(D_{2n})} \sqrt{(2\mathrm{n}-1-\frac{2m}{n})^2+(\mathrm{n}+1-\frac{2m}{n})^2}$$
$$= (n-1)\sqrt{(2n-1-\frac{2m}{n})^2+(\mathrm{n}+1-\frac{2m}{n})^2}.$$

- In the second case, the relationship between the identity element and the reflection elements is calculated

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_2 \in E\,\Gamma(D_{2n})} \sqrt{(d_e-\frac{2m}{n})^2+(d_{P_2}-\frac{2m}{n})^2}$$
$$= \sum_{e,P_2 \in E\,\Gamma(D_{2n})} \sqrt{\left(2n-1-\frac{2m}{n}\right)^2+\left(n-\frac{2m}{n}\right)^2}.$$

Since *e* is adjacent to every vertices of P_2, and $|P_2| = n$, then

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = \sum_{e,P_2 \in E\,\Gamma(D_{2n})} \sqrt{(2n-1-\frac{2m}{n})^2+(n-\frac{2m}{n})^2}$$
$$= n\sqrt{\left(2n-1-\frac{2m}{n}\right)^2+\left(n-\frac{2m}{n}\right)^2}$$

- And finally, the relationship between the rotation element and the reflection element is calculated

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = \sum_{\substack{,u\in P_1 \\ v\in P_2},P_1,P_2 \in E\,\Gamma(D_{2n})} \sqrt{\left(d_u-\frac{2m}{n}\right)^2+\left(d_v-\frac{2m}{n}\right)^2}$$
$$= \sum_{\substack{,u\in P_1 \\ v\in P_2}} \sqrt{\left(n+1-\frac{2m}{n}\right)^2+\left(n-\frac{2m}{n}\right)^2}.$$

Since P_1 is adjacent to every vertices of P_2, $|P_1| = n - 1$ and $|P_2| = n$, then

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = \sum_{\substack{,u\in P_1\\ v\in P_2}} \sqrt{\left(n+1-\frac{2m}{n}\right)^2 + \left(n-\frac{2m}{n}\right)^2}$$

$$= n(n-1)\sqrt{\left(n+1-\frac{2m}{n}\right)^2 + \left(n-\frac{2m}{n}\right)^2}$$

Based on the three cases above, in general, the average Sombor index of a coprime graph $\Gamma_{D_{2n}}$ is:

$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{\left(2n-1-\frac{2m}{n}\right)^2 + \left(\mathrm{n}+1-\frac{2m}{n}\right)^2} + n\sqrt{\left(2n-1-\frac{2m}{n}\right)^2 + (n-\frac{2m}{n})^2}$$

$$+n(n-1)\sqrt{\left(n+1-\frac{2m}{n}\right)^2 + \left(n-\frac{2m}{n}\right)^2}. \blacksquare$$

Conclusion

In this research, several conclusions were obtained regarding the Sombor index, reduced Sombor index, and average Sombor index on the coprime graph of the dihedral group D_{2n} for n is a odd prime power.

1. The Sombor index of the coprime graph of the dihedral group D_{2n} is
$$SO\ (G) = (n-1)\sqrt{(2n-1)^2 + (n-1)^2} + (n)\sqrt{(2n-1)^2 + (n)^2}$$
$$+n(n-1)\sqrt{(n+1)^2 + (n)^2}$$
2. The reduced Sombor index of the coprime graph of the dihedral group D_{2n} is
$$SO_{red}\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{(2n-2)^2 + n^2} + n\sqrt{(2n-2)^2 + (n-1)^2}$$
$$+n(n-1)\sqrt{n^2 + (n-1)^2}$$
3. The average Sombor index of the coprime graph of the dihedral group D_{2n} is
$$SO_{avr}\left(\Gamma_{D_{2n}}\right) = (n-1)\sqrt{\left(2n-1-\frac{2m}{n}\right)^2 + \left(\mathrm{n}+1-\frac{2m}{n}\right)^2}$$
$$+n\sqrt{\left(2n-1-\frac{2m}{n}\right)^2 + (n-\frac{2m}{n})^2}$$
$$+n(n-1)\sqrt{\left(\mathrm{n}+1-\frac{2m}{n}\right)^2 + \left(\mathrm{n}-\frac{2m}{n}\right)^2}$$

Thus, general formulas for the Sombor index, reduced Sombor index, and average Sombor index on coprime graphs of the dihedral group D_{2n} have been obtained for n is a odd prime power.

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