Proceedings of Science and Mathematics
Faculty of Science, Universiti Teknologi Malaysia

Volume 34, 2026, page 1-6

Albertson Index and Forgotten Index on the Power Graph of Dihedral Groups (D_{2n}) of Prime Power Order

Gusti Yogananda Karang[a], I Gede Adhitya Wisnu Wardhana[a], Rio Satriyantara[a]*
[a]Department of Mathematics, University of Mataram, Lombok, Indonesia
*Corresponding author: riosatriyantara@staff.unram.ac.id

Abstract
Graph theory plays an important role in the study of algebraic structures through graphical representations, one of which is the power graph of a group. In this paper, we investigate degree-based topological indices, namely the Albertson index and the forgotten index, on the power graph of the dihedral group D_{2n}. The power graph is constructed by connecting two distinct elements of D_{2n} whenever one is a power of the other. By utilizing the structural properties of rotations and reflections in dihedral groups, we determine the relevant graph characteristics, including the degree of each vertex. Furthermore, general formulas for the Albertson index and the forgotten index of the power graph of D_{2n} are derived. The results reveal the relationship between the algebraic properties of dihedral groups and degree-based graph invariants, and are expected to contribute to the study of topological indices on graphs associated with algebraic structures.
Keywords: Power graph; dihedral group; Albertson index; forgotten index; graph topological indices

Introduction

Graph theory is a branch of mathematics that studies relationships between vertices and edges in graphs. Many studies on graphs focus on their representations in various fields, one of which is algebraic structures such as groups. These groups can be represented by different types of graphs, including coprime graphs, non-prime graphs, power graphs, and intersection graphs [1]. A graph is said to be complete if every vertex in the graph is adjacent to all other vertices [2]. This study investigates graph representations arising from algebraic structures, particularly dihedral groups, specifically focusing on dihedral groups. A dihedral group is a group consisting of a set of elements that includes rotation elements $\{a\}$ and reflection elements $\{b\}$ of a regular polygon with n sides [3].

Dihedral groups can be represented by several types of graphs, such as coprime graphs, non-coprime graphs, power graphs, and unit graphs. In particular, this research focuses on power graphs. Previous research in [4] studied the power graph of the integer modulo group, where the graph represents the algebraic relationship between two elements of a group. Meanwhile, using the same type of graph, the study in [5] investigated the dihedral group and showed that its power graph is connected, consisting of a complete subgraph and a star subgraph.

In graph theory, there is a concept called topological indices. A topological index is a numerical value that represents the structural properties and connectivity of a graph [6]. Topological indices are used to numerically represent chemical structures and to predict chemical properties, molecular physical structures, and chemical reactions [7, 8]. This study focuses on investigating the topological indices of graphs, specifically the Forgotten index and the Albertson index, derived from the power graph representation of the dihedral group. The research in [9] an index is defined based on a degree-based graph invariant that did not attract attention in the mathematical chemistry literature for more than 40 years. Therefore, this index is called the Forgotten topological index. The study in [10] computed the Forgotten index of the sub-division-related generalized F-sum graphs based on strong product. The Albertson index was first introduced by Albertson in his paper using the concept of the imbalance of the edge from graph [11]. The study in [12] develops the Albertson index into the modified Albertson index.

This study presents general formulas for the Albertson index and Forgotten index of the power graph of dihedral groups D_{2n} where n is prime power.

Preliminaries

This study is conducted using a theoretical approach based on literature review and mathematical derivation. The research begins with a literature review, followed by deriving the general formula for the Forgotten index and the Albertson index of power graph on dihedral group, generalized for several cases of n. Subsequently, a conjecture is formulated, and the conjecture is proven. If the conjecture is validated, it is then established as a theorem.

The dihedral group is the group of symmetries of a regular polygon with n sides, consisting of rotation elements $\{a\}$ and reflection elements $\{b\}$.

Definition 1. [3] The group representation of the dihedral groups is expressed as:

$$D_{2n} = \{\langle a, b\rangle | a^n = b^2 = e, a^{-1} = bab^{-1}\}, n \in \mathbb{N}, n \neq 1, 2.$$

The power graph of a group is defined as a graph whose vertices consist of all the elements of the group, and two distinct vertices are adjacent if one element is a power of the other.

Definition 2. [3] Suppose G is a group. The set of vertices $V(G)$ is equal to G and two distinct vertices, $a, b \in G$ are neighbors if and only if $a = b^x$ or $a^y = b$, for some $x, y \in \mathbb{N}$.

The power graph of dihedral group is stated as follows:

Theorem 1. [5] Let D_{2n} a dihedral group. If $n = p^k$, where p is prime numbers and $k \in \mathbb{N}$, the power graph of D_{2n} consists of two non-disjoint subgraphs, namely a complete subgraph and a star subgraph.

Here are some examples of the power graph representations of the dihedral group with the order of powers of primes.

Example 1. Given that the group $D_{2.3} = \{e, a, a^2, b, ab, a^2b\}$.

Table 1: The neighbourhood table of D_6

Elements of $\boldsymbol{D_6}$	$\boldsymbol{e}$	$\boldsymbol{a}$	$\boldsymbol{a^2}$	$\boldsymbol{b}$	$\boldsymbol{ab}$	$\boldsymbol{a^2b}$
$\boldsymbol{e}$	-	$e = a^3$	$e = (a^2)^3$	$e = b^2$	$e = (ab)^2$	$e = (a^2b)^2$
$\boldsymbol{a}$	$e = a^3$	-	$a = (a^2)^2$	-	-	-
$\boldsymbol{a^2}$	$e = (a^2)^3$	$a = (a^2)^2$	-	-	-	-
$\boldsymbol{b}$	$e = b^2$	-	-	-	-	-
$\boldsymbol{ab}$	$e = (ab)^2$	-	-	-	-	-
$\boldsymbol{a^2b}$	$e = (a^2b)^2$	-	-	-	-	-

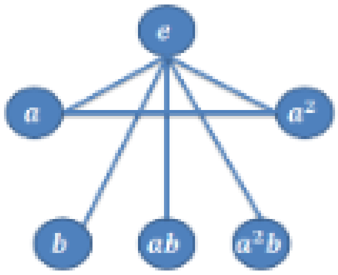

Figure 1 Power graph of the dihedral group D_6

A fundamental concept in graph theory is the degree of a vertex, which describes the number of edges incident to a vertex.

Definition 3. [4] Let Γ be a graph, where $V(\Gamma)$ represents the set of vertices. The degree of $v_i \in V(\Gamma)$ is defined as the number of edges connected to v_i, denoted by d_{v_i}.

The degree of each vertex of the power graph of the dihedral group is given in the following theorem.

Theorem 2 [5] The degree of each vertex of the power graph of the dihedral groups D_{2n} as follows:

i) $d_e = 2n - 1$,
ii) $d_{a^i} = n - 1$, for all $i \in \mathbb{Z}$, $1 \leq i \leq n - 1$,
iii) $d_{a^j b} = 1$, for all $j \in \mathbb{Z}$, $0 \leq j \leq n - 1$.

The forgotten index and the Albertson index of graph are defined as follows:

Definition 4. [9] Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \ldots, v_n\}$ is the vertex set of the graph Γ, then Forgotten index of a Γ, denoted as $F(\Gamma)$ is defined as follows:

$$F(\Gamma) = \sum_{v_i \in V(\Gamma)} d_{v_i}^3.$$

Definition 5. [9] Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \ldots, v_n\}$ is the vertex set and $E(\Gamma) = \{u_1v_1, u_2v_2, \ldots, u_nv_n\}$ is the edge set of graph Γ, then the Albertson index of a Γ, denoted as $A(\Gamma)$ is defined as follows:

$$A(\Gamma) = \sum_{uv \in E(\Gamma)} |d_u - d_v|.$$

Results and discussion

In this section, we derive the forgotten index and the Albertson index of the power graph on dihedral group with order p^k, p prime numbers, $k \in \mathbb{N}$.

Based on the Theorem 1, general formulas related to the Albertson index and the forgotten index are derived, as presented in Theorem 3 and Theorem 4 in the following.

Theorem 3. Let Γ be a power graph of dihedral group D_{2n} with $n = p^k$, where p is a prime number and $k \in \mathbb{N}$, then the forgotten index of $\Gamma_{D_{2p^k}}$ is

$$F\left(\Gamma_{D_{2p^k}}\right) = p^{4k} + 4p^{3k} - 6p^{2k} + 3p^k.$$

Proof.

$$\begin{aligned} F\left(\Gamma_{D_{2p^k}}\right) &= \sum_{v_i \in V(\Gamma)} d_{v_i}^3 \\ &= d_e^3 + \underbrace{d_a^3 + d_{a^2}^3 + \cdots + d_{a^{p^k-1}}^3}_{p^k-1} + \underbrace{d_b^3 + d_{ab}^3 + d_{a^2b}^3 + \cdots + d_{a^{p^k-1}b}^3}_{p^k} \end{aligned}$$

Based on Theorem 2,

$$\begin{aligned} F\left(\Gamma_{D_{2p^k}}\right) &= \left(2p^k - 1\right)^3 + \underbrace{\left(p^k - 1\right)^3 + \left(p^k - 1\right) + \cdots + \left(p^k - 1\right)}_{p^k-1} + \underbrace{1^3 + 1^3 + 1^3 + \cdots + 1^3}_{p^k} \\ &= \left(2p^k - 1\right)^3 + \left(p^k - 1\right)\left(p^k - 1\right)^3 + p^k \\ &= \left(8p^{3k} - 12p^{2k} + 6p^k - 1\right) + \left(p^{4k} - 4p^{3k} + 6p^{2k} - 4p^k + 1\right) + p^k \\ &= p^{4k} + 4p^{3k} - 6p^{2k} + 3p^k. \end{aligned}$$

Theorem 4. Let Γ be a power graph of dihedral group D_{2n} with $n = p^k$, where p is a prime number and $k \in \mathbb{N}$, then the Albertson index of $\Gamma_{D_{2p^k}}$ is

$$A\left(\Gamma_{D_{2p^k}}\right) = 3p^k\left(p^k - 1\right).$$

Proof. Based on Theorem 1. The graph of $\Gamma_{D_{2p^k}}$ is partitioned into two types as follows:

i) Complete subgraph, with vertex is $V_1\left(\Gamma_{D_{2p^k}}\right) = \left\{e, a, a^2, \dots, a^{p^k-1}\right\}$ in which every vertex is adjacent.

ii) Star subgraph, with vertex set is partitioned into $V_2\left(\Gamma_{D_{2p^k}}\right) = \{e\}$ and $V_3\left(\Gamma_{D_{2p^k}}\right) = \left\{b, ab, a^2b, \dots, a^{p^k-1}b\right\}$ in which each vertex in $V_3\left(\Gamma_{D_{2p^k}}\right)$ is adjacent only to vertices in $V_2\left(\Gamma_{D_{2p^k}}\right)$.

Based on the adjacency of vertices in $\Gamma_{D_{2p^k}}$, the Albertson index of $\Gamma_{D_{2p^k}}$ is

$$\begin{aligned} A\left(\Gamma_{D_{2p^k}}\right) &= \sum_{uv \in E(\Gamma)} |d_u - d_v| \\ &= \sum_{ea^i \in E(\Gamma)} \left|d_e - d_{a^i}\right| + \sum_{a^j a^i b \in E(\Gamma)} \left|d_{a^j} - d_{a^i}\right| + \sum_{ea^i b \in E(\Gamma)} \left|d_e - d_{a^i b}\right|. \end{aligned}$$

Based on Theorem 2,

$$A\left(\Gamma_{D_{2p^k}}\right) = \underbrace{\left|\left(2p^k - 1\right) - \left(p^k - 1\right)\right| + \cdots + \left|\left(2p^k - 1\right) - \left(p^k - 1\right)\right|}_{n-1}$$

$$
\begin{aligned}
&+\underbrace{\left|\left(p^k-1\right)-\left(p^k-1\right)\right|+\cdots+\left|\left(p^k-1\right)-\left(p^k-1\right)\right|}_{\frac{p^{2k}-3p^k+2}{2}} \\
&+\underbrace{\left|\left(2p^k-1\right)-1\right|+\cdots+\left|\left(2p^k-1\right)-1\right|}_{p^k} \\
&=\left(p^k-1\right)\left(p^k\right)+\left(\frac{p^{2k}-3p^k+2}{2}\right)0+p^k\left(2p^k-2\right) \\
&=p^{2k}-p^k+2p^{2k}-2p^k \\
&=3p^{2k}-3p^k \\
&=3p^k\left(p^k-1\right).
\end{aligned}
$$

Conclusion

This paper investigates the forgotten index and the Albertson index of the power graph associated with dihedral groups whose parameter is a prime power. By analyzing the structural properties of the power graph, which consists of a complete subgraph formed by rotation elements and a star subgraph formed by reflection elements, the vertex degrees of the graph can be determined systematically. Based on these structural characteristics, general formulas for the forgotten index and the Albertson index are obtained. The results show that the group parameter expressed as a prime power plays an important role in determining the degree distribution and consequently influences the values of these degree-based topological indices. These findings contribute to the study of topological indices on graphs derived from algebraic structures and provide a foundation for further investigations of other graph invariants on power graphs of finite groups.

Acknowledgement

The author would like to express their sincere gratitude to their colleagues and supervisors for the valuable feedback and guidance that greatly contributed to the completion of this study.

References

[1] Devandra, U. &. (2022). Mendeskripsikan grup menggunakan berbagai graf. *UJMC (Unisda Journal of Mathematics and Computer Science), 8*(1), 27-34.

[2] Aulia, S. A., Wardhana, I. G., Irwansyah, Salwa, Misuki, W. U., & Nghiem, N. D. (2023). The structures of non-coprime graphs for finite groups from dihedral groups with regular composite orders. *InPrime: Indonesian Journal of Pure and Applied Mathematics, 5*(2), 115-122.

[3] Syarifudin, A. G., Wardhana, I. G., Switrayni, N. W., & Aini, Q. (2021). The clique numbers and chromatic numbers of the coprime graph of a dihedral group. *IOP Conference Series: Materials Science and Engineering, 1115*(1).

[4] Asmarani, E. Y., Lestari, S. T., Purnamasari, D., Syarifudin, A. G., Salwa, S., & Wardhana, I. G. A. W. (2023). The first Zagreb index, the Wiener index, and the Gutman index of the power of dihedral group. *CAUCHY: Jurnal Matematika Murni dan Aplikasi*, 7(4), 513-520.

[5] Putra, L. R., Awanis, Z. Y., Salwa, Aini, Q., & Wardhana, I. G. (2023). The power graph representation for integer modulo. *BAREKENG: Journal of Mathematics and Its Applications, 17*(3), 1393–1400.

[6] Aykaç, S., Akgüneş, N., & Çevik, A. S. (2019). Analysis of Zagreb indices over zero-divisor graphs of commutative rings. *Asian-European Journal of Mathematics, 12*(06).

[7] Bolombias, M. H., Putra, G. L., & Haning, F. O. (2024). Indeks topologi pada graf pembagi nol. *Jurnal Riset dan Aplikasi Matematika (JRAM), 08*(02), 105 - 121.

[8] Semil @ Ismail, G., Sarmin, N. H., Alimon, N. I., & Maulana, F. (2023). The first Zagreb index of the zero divisor graph for the ring of integers modulo power of primes. *Malaysian Journal of Fundamental and Applied Sciences, 19*(5), 892 - 900.

[9] Gutman, I., Ghalavand, A., Dehghan-Zadeh, T., & Ashrafi, a. A. (2017). Graphs with Smallest forgotten index. *Iranian Journal of Mathematical Chemistry, 8*(3), 259 - 273.

[10] Javaid, M., Javed, S., Memon, S. Q., & Alanazi, A. M. (2021). Forgotten index of generalized operations on graphs. *Journal of Chemistry, 2021*(1), 1-14.

[11] Albertson, M. O. (1997). The irregularity of a Graph. *Ars Combinatoria, 46*, 219-226.

[12] Yousaf, S., Bhatti, A. A., & Ali, A. (2019). A note on the modified Albertson index. *arXiv preprint arXiv, 1902.01809*.

[13] Juliana, R., Masriani, Wardhana, I. G., Switrayni, N. W., & Irwansyah. (2020). Coprime graph of integer modulo n group and its subgroups. *Journal Of Fundamental Mathematicsand Applications (JFMA), 3*(1), 15-18.