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Inverse Degree and Forgotten Indices of the Non-Commuting Graph of the Group U_{6n}

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Abstract
Graph theory plays an important role in studying the structural properties of algebraic systems through graph representations. Various degree–based topological indices have been widely used to analyze such graphs due to their ability to capture structural characteristics. This article introduces a new approach to computing the inverse degree index and the forgotten index of non-commuting graphs, with particular emphasis on the group U_{6n}. The inverse degree index was first introduced by Siemion Fajtlowicz, where it was explicitly defined as $ID(G) = \sum_{v \in V(G)} \frac{1}{deg(v)}$. Meanwhile, the forgotten index of a group G is defined as $F(G) = \sum_{v \in V(G)} deg(v)^3$. Both indices are degree-based topological indices and are closely related to the well-known Zagreb indices in graph theory. Studies on non-commuting graphs of the group U_{6n} are still relatively limited. Therefore, this research aims to connect the structure of such graphs with degree-based topological indices, particularly the inverse degree index and the forgotten index. The main focus of this study is the computation of these two indices for the non-commuting graph of the group U_{6n}. Based on the analysis carried out, explicit formulas of the inverse degree index and the forgotten index of the non-commuting graph for the group U_{6n} are obtained.
Keywords: inverse degree index; forgotten index; non-commuting graph; group U_{6n}

Introduction

Graph theory is one of the branches of mathematics that has developed very rapidly and has strong connections with other scientific disciplines. In its applications, graph theory is widely used in other areas such as scheduling, chemistry, biological systems, global economic and financial systems, computer science, and industry [1,2]. A graph Γ is defined as a mathematical structure consisting of two sets, namely the vertex set, $V(\Gamma)$ and the edge set, $E(\Gamma)$ [3]. This research focuses on non-commuting graph, which is the complement of commuting graph. In a non-commuting graph, two elements are said to be adjacent if and only if they do not commute. Recent research in 2024 on the non-commuting graph of the group U_{6n} investigated the detour index and the eccentric connectivity index of $\Gamma(U_{6n})$ [4].

The study of the non-commuting graphs can be viewed as part of the development of topological indices. In chemistry, topological indices derived from molecular graphs are used as structural descriptors to model the physicochemical properties of compounds by representing the connectivity between atoms in a molecule [5]. Among these indices, the inverse degree index has attracted considerable attention and was introduced in 1988 [6]. In addition, the forgotten index was introduced in 2015 and has been widely used to predict the physicochemical properties of molecular structures [7]. However, there is still limited research on the computation of these indices for non-commuting graphs of algebraic structures, particularly for the group U_{6n}. Therefore, this study aims to determine and analyze the inverse degree index and the forgotten index for the non-commuting graph of the group U_{6n}.

Methods

The stages of this research were carried out using a theoretical approach that includes the collection and review of literature related to graphs, groups, and topological indices. Subsequently, the structure of the non-commuting graph of the group U_{6n} was analyzed based on case patterns obtained from related studies. Based on this analysis, conjectures regarding the characteristics of the inverse degree index and the forgotten index were formulated and then proven mathematically so that they could be established as theorems. The conclusions of the study were drawn based on the results obtained.

Before discussing the main results, several definitions and lemmas required for the discussion are presented.

Definition 1. [4] Let U_{6n} be a group of order $6n$ defined by $U_{6n} = \langle x, y \mid x^{2n} = y^3 = e = 1,\ x^{-1}yx = y^{-1} \rangle$, for $n \geq 1$, with the center of the group given by $Z(U_{6n}) = \langle x^2 \rangle$.

Definition 2. [8] Let $Z(G)$ be the center of a finite non-abelian group G. The non-commuting graph, $\Gamma(G)$ is a graph whose vertices are the non-central elements of G, denoted by $G \setminus Z(G)$. Two vertices x and y are adjacent if and only if $xy \neq yx$, where $x, y \in G \setminus Z(G)$.

Definition 3. [6] The inverse degree index of a molecular graph G, denoted by $ID(G)$, is defined as

$$ID(G) = \sum_{v \in V(G)} \frac{1}{deg(v)},$$

where $deg(v)$ denotes the degree of the vertex v in the graph G.

Definition 4. [7] The forgotten index of a molecular graph G, denoted by $F(G)$, is defined as

$$F(G) = \sum_{v \in V(G)} deg(v)^3,$$

where $deg(v)$ denotes the degree of the vertex v in the graph G.

Lemma 1. [4] Let $n \geq 1$ be an integer and $\Gamma = \Gamma(U_{6n})$. Then, for $0 \leq r \leq n-1$ and $k = 1, 2$, we have: $deg(x^{2r+1}) = 4n,\ deg(x^{2r+1}y^k) = 4n$, and $deg(x^{2r}y^k) = 3n$.

Results and discussion

In this section, the forgotten and the inverse degree indices of the non-commuting graph for the group U_{6n} are presented. Based on the lemma previously obtained for the group U_{6n}, several general properties of the associated non-commuting graph can be derived. These results are presented in Theorem 1 as follows.

Theorem 1. Let $n \geq 1$ be a positive integer and consider the non-commuting graph associated with the group U_{6n}. For each r such that $0 \leq r \leq n-1$ and for $k = 1, 2$, it is obtained that:

1. The vertex (x^{2r+1}) appears n times;

2. The vertex $(x^{2r+1}y^k)$ appears $2n$ times;
3. The vertex $(x^{2r}y^k)$ appears $2n$ times.

Proof. From the structure of the group U_{6n} with order $6n$, we can observe the pattern of its elements for all r satisfying $0 \leq r \leq n-1$, as follows:

1. The elements (x^{2r}) form the set $\{e, x^2, x^4, ...\}$ consisting of n elements.
2. The elements (x^{2r+1}) form the set $\{x, x^3, x^5, ...\}$ consisting of n elements.
3. The elements $(x^{2r}y)$ form the set $\{x^2y, x^4y, x^6y, ...\}$ consisting of n elements.
4. The elements $(x^{2r+1}y)$ form the set $\{xy, x^3y, x^5y, ...\}$ consisting of n elements.
5. The elements $(x^{2r}y^2)$ form the set $\{x^2y^2, x^4y^2, x^6y^2, ...\}$ consisting of n elements.
6. The elements $(x^{2r+1}y^2)$ form the set $\{xy^2, x^3y^2, x^5y^2, ...\}$ consisting of n elements.

Therefore, since the center of U_{6n} is $Z(U_{6n}) = \langle x^{2r} \rangle$, which contains n elements, the following hold with $k = 1, 2$:

1. The vertex of x^{2r+1} appears n times.
2. The vertex of $x^{2r+1}y^k$ appears $2n$ times.
3. The vertex of $x^{2r}y^k$ appears $2n$ times. ∎

Based on the proof of Theorem 1, the general formulas related to the inverse degree index and the forgotten index of the non-commuting graph of the group U_{6n} can be derived, as presented in Theorem 2 and Theorem 3 below.

Theorem 2. Let the non-commuting graph of the group U_{6n} be given, with $n \geq 1$. Then, the inverse degree index of the non-commuting graph of the group U_{6n} is $ID(U_{6n}) = \frac{17}{12}$.

Proof.

$$\begin{aligned} ID(U_{6n}) &= \sum_{v \in V(U_{6n})} \frac{1}{\deg(v)} \\ &= n \cdot \left(\frac{1}{4n}\right) + 2n \cdot \left(\frac{1}{4n}\right) + 2n \cdot \left(\frac{1}{3n}\right) \\ &= \frac{1}{4} + \frac{1}{2} + \frac{2}{3} \\ &= \frac{17}{12} \end{aligned}$$

∎

Theorem 3. Let the non-commuting graph of the group U_{6n} be given, with $n \geq 1$. Then, the forgotten index of the non-commuting graph of the group U_{6n} is $F(U_{6n}) = 246n^4$.

Proof.

$$\begin{aligned} F(U_{6n}) &= \sum_{v \in V(U_{6n})} \deg(v)^3 \\ &= n \cdot (4n)^3 + 2n \cdot (4n)^3 + 2n \cdot (3n)^3 \\ &= 64n^4 + 128n^4 + 54n^4 \end{aligned}$$

$$= 246n^4$$

■

After obtaining Theorem 2 and Theorem 3, which provide the general formulas for the inverse degree index and the forgotten index of the non-commuting graph of the group U_{6n}, an example case is presented for a simple value of $n = 3$. This example is used to illustrate the structure of the non-commuting graph and to provide a clearer visualization of the adjacency relationships among the vertices involved in the computation of the indices. The non-commuting graph of the group U_{6n} for $n = 3$ is shown in **Figure 1**.

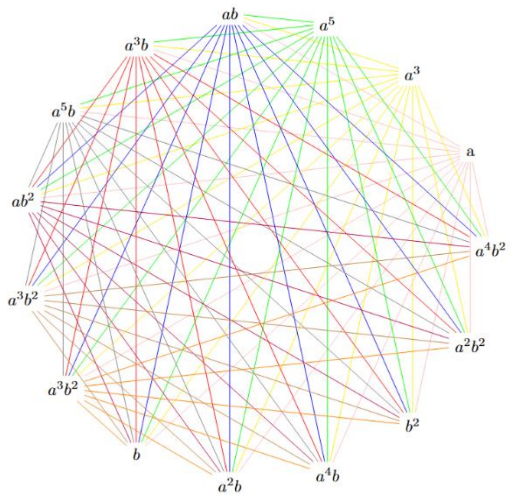

Figure 1 Graph $\Gamma(U_{6\cdot3})$

Example 1. Consider the non-commuting graph of the group U_{6n} for $n = 3$, as illustrated in **Figure 1**. We compute the inverse degree index of the graph $\Gamma(U_{6n})$.
Based on the definition of the inverse degree index,

$$ID(G) = \sum_{v \in V(G)} \frac{1}{deg(v)}.$$

From Figure 1, the vertices of the non-commuting graph $\Gamma(U_{6\cdot3})$ can be grouped according to their degrees as follows:

- $3n$ vertices with degree $4n$,
- $2n$ vertices with degree $4n$,
- $2n$ vertices with degree $3n$.

Thus,

$$ID(U_{6\cdot3}) = \sum_{v \in V(U_{6\cdot3})} \frac{1}{deg\ deg\ (v)} = (3) \cdot \left(\frac{1}{4(3)}\right) + 2(3) \cdot \left(\frac{1}{4(3)}\right) + 2(3) \cdot \left(\frac{1}{3(3)}\right)$$
$$= \frac{1}{4} + \frac{1}{2} + \frac{2}{3}$$
$$= \frac{17}{12}.$$

Therefore, the inverse degree index of the non-commuting graph of U_{6n} for $n = 3$ is $ID(U_{6\cdot 3}) = \frac{17}{12}$.

Example 2. Consider the non-commuting graph of the group U_{6n} for $n = 3$, as illustrated in **Figure 1**. We compute the forgotten index of the graph $\Gamma(U_{6n})$.
Based on the definition of the forgotten index,

$$F(G) = \sum_{v \in V(G)} deg\,(v)^3.$$

From Figure 1, the vertices of the non-commuting graph $\Gamma(U_{6\cdot 3})$ can be grouped according to their degrees as follows:

- $3n$ vertices with degree $4n$,
- $2n$ vertices with degree $4n$,
- $2n$ vertices with degree $3n$.

Thus,

$$\begin{aligned} F(U_{6\cdot 3}) &= \sum_{v \in V(U_{6\cdot 3})} \deg\,(v)^3 \\ &= (3) \cdot \big(4(3)\big)^3 + 2(3) \cdot \big(4(3)\big)^3 + 2(3) \cdot \big(3(3)\big)^3 \\ &= 5184 + 10368 + 4374 \\ &= 19926 \\ &= 246(3)^4. \end{aligned}$$

Therefore, the forgotten index of the non-commuting graph of U_{6n} for $n = 3$ is $F(U_{6\cdot 3}) = 246(3)^4$.

Conclusion

This study successfully derives explicit formulas for the inverse degree index and the forgotten index of the non-commuting graph associated with the group U_{6n}. By utilizing the structural properties of the vertex degrees of the group U_{6n}, both indices are expressed in terms of the parameter n. The obtained results show that $ID(U_{6n}) = \frac{17}{12}$ for the inverse degree index and $F(U_{6n}) = 246n^4$ for the forgotten index. This study contributes to the development of degree-based topological indices of the non-commuting graphs.

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