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Topological Indices of the Power Graph of Modular Integer Groups

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Abstract

Topological indices are graph invariants that are widely used to represent the structural characteristics of graphs through relationships among vertices, such as vertex degrees and adjacency. These indices play an important role in measuring structural complexity and comparing different graph structures. This study focuses on the analysis of topological indices that depend on the structural properties of graphs. Then, the discussion is then applied to graphs constructed from algebraic structures, namely power graphs of modular integer groups. A power graph is defined based on the power relation among elements of a modular group, so that the resulting graph structure reflects the underlying algebraic properties of the group. This study contributes the development of topological index theory for graphs arising from algebraic structures, particularly power graphs of modular integer groups.

Keywords: Topological index, power graph, modular integer group, algebraic graph theory

Introduction

Graph theory was first introduced by the mathematician Leonhard Euler in 1736. Graph theory is one of the branches of mathematics that has developed rapidly and has wide applications in various fields, such as computer science, chemistry, biology, and pure mathematics. Moreover, in the context of pure mathematics, graphs are often used as a representational tool to describe the structure of a group [1]. The combination of graph theory and group theory has become one of the most important approaches in pure mathematics.

In mathematics, there are various types of graphs that can be used to represent different group structures, including inverse graphs, identity graphs, coprime graphs, non-coprime graphs, commuting graphs, non-commuting graphs, and several other types of graphs [2]. These graphs are used to study group properties through a graph-theoretic approach; therefore, graph theory and group theory are frequently combined in many studies.

Graph theory can also be applied to represent various algebraic systems, such as groups, rings, and modules [3]. Several studies have discussed graph representations in the context of groups, for example, coprime and non-coprime graphs of dihedral groups, prime coprime graphs of the group of integers modulo n , power graphs of dihedral groups, and coprime graphs of the group of integers modulo n with prime power order [4,5,6].

One of the most widely studied graph representations is the power graph. A power graph is a graph whose vertices represent all elements of a group, and two vertices are said to be adjacent if one vertex can be expressed as a power of the other [6]. A further application of graph theory in mathematics is in determining topological indices. Several researchers have investigated various topological indices, including the harmonic index, Zagreb index, Gutman index, and Wiener index, of coprime graphs and power graphs derived from groups of integers modulo n or dihedral groups [2].

Although numerous studies have examined topological indices on various types of graphs derived from algebraic structures, research on topological indices of power graphs of the group of integers modulo n remains relatively limited. Therefore, this study aims to determine and analyze several topological indices of power graphs of the group of integers modulo n and to investigate the properties of the resulting graphs.

Definition 1 [7] The power graph of a group G is defined as a graph whose vertex set consists of all elements of the group G , and two distinct vertices a and b are adjacent if and only if $a^x = b$ or $b^y = a$ for some positive integers x and y .

Definition 2 [8] The group G , called the group of integers modulo n , is a group consisting of the set of all non-negative integers equipped with addition modulo n . It is denoted by $\mathbb{Z}_n = \{0, 1, 2, \dots, n - 1\}$ with the operation $+_{mod(n)}$.

Definition 3 [9] The atom-bond connectivity (ABC) index of a graph, written as $ABC(G)$, quantifies the contributions of every connected vertex pair u and v , is formulated as:

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{deg(u) + deg(v) - 2}{deg(u)deg(v)}}$$

where $deg(u)$ and $deg(v)$ denoted the degrees of vertices u and v , respectively.

Definition 4 [10] The Nirmala index of a graph G , denoted as $N(G)$ is determined by:

$$N(G) = \sum_{uv \in E(G)} \sqrt{deg(u) + deg(v)}$$

where $deg(u)$ and $deg(v)$ are the degrees of adjacent vertices u and v .

Definition 5 [11] Let G be a graph. The Forgotten index of G denoted by $F(G)$, is defined as:

$$F(G) = \sum_{v \in V(G)} deg(v)^3$$

Where $deg(v)$ is the degree of vertex v , for all vertex in G .

Lemma 1 [12] Let \mathbb{Z}_n be the group of integers modulo n equipped with the operation $+_{mod(n)}$. If $n = p^k$, where $k \in \mathbb{N}$ and p is a prime number, then the power graph constructed from the group \mathbb{Z}_n denoted by $\Gamma_{\mathbb{Z}_n}$, is a complete graph K_n .

Results and discussion

Theorem 1 Let $\Gamma_{\mathbb{Z}_n}$ be the power graph of the group of integers modulo n , where $n = p^k$ for some $k \in \mathbb{N}$ and p is a prime number. Then the ABC index of $\Gamma_{\mathbb{Z}_n}$ is given by

$$ABC(\Gamma_{\mathbb{Z}_n}) = \frac{p^k(p^k - 1)}{2} \sqrt{\frac{2p^k - 4}{p^{2k} - 2p^k + 1}}$$

Proof Based on Lemma 1, which states that $\Gamma_{\mathbb{Z}_n}$ is a complete graph, the number of its edges is $\binom{n}{2}$.

Since $n = p^k$, $k \in \mathbb{N}$ and p is a prime number, this can be written as $\binom{p^k}{2}$, or

$$\begin{aligned} C_2^{p^k} &= \frac{p^k!}{(p^k - 2)! 2!} \\ &= \frac{p^k(p^k - 1)(p^k - 2)!}{(p^k - 2)! 2!} \\ &= \frac{p^k(p^k - 1)}{2}. \end{aligned}$$

Each vertex has degree $p^k - 1$, Therefore, the ABC index of the power graph of the group of integers modulo n is given by

$$\begin{aligned} ABC(\Gamma_{\mathbb{Z}_n}) &= \sum_{uv \in E(G\Gamma_{\mathbb{Z}_n})} \sqrt{\frac{deg(u) + deg(v) - 2}{deg(u)deg(v)}} \\ &= \frac{p^k!}{(p^k - 2)! 2!} \sqrt{\frac{(p^k - 1) + (p^k - 1) - 2}{(p^k - 1)(p^k - 1)}} \\ &= \frac{p^k(p^k - 1)}{2} \sqrt{\frac{2p^k - 4}{p^{2k} - 2p^k + 1}} \end{aligned}$$

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Theorem 2 Let $\Gamma_{\mathbb{Z}_n}$ be the power graph of the group of integers modulo n , where $n = p^k$ for some $k \in \mathbb{N}$ and p is a prime number. Then, the Nirmala index of $\Gamma_{\mathbb{Z}_n}$ is given by

$$N(\Gamma_{\mathbb{Z}_n}) = \frac{p^{2k} - p^k}{2} \sqrt{2p^k - 2}.$$

Proof Based on Lemma 1, which states that $\Gamma_{\mathbb{Z}_n}$ is a complete graph, the number of vertices is n where $n = p^k$, $k \in \mathbb{N}$ and p is a prime number. Each vertex has degree $p^k - 1$, Therefore, the Nirmala index of the power graph of the group of integers modulo n is given by

$$\begin{aligned} N(\Gamma_{\mathbb{Z}_n}) &= \sum_{uv \in E(\Gamma_{\mathbb{Z}_n})} \sqrt{deg(u) + deg(v)} \\ &= \frac{p^k!}{(p^k - 2)! 2!} \sqrt{(p^k - 1) + (p^k - 1)} \\ &= \frac{p^{2k} - p^k}{2} \sqrt{2p^k - 2}. \end{aligned}$$

■

Theorem 3 Let $\Gamma_{\mathbb{Z}_n}$ be the power graph of the group of integers modulo n , where $n = p^k$ for some $k \in \mathbb{N}$ and p is a prime number. Then, the Forgotten index of $\Gamma_{\mathbb{Z}_n}$ is given by

$$F(G\Gamma_{\mathbb{Z}_n}) = p^{4k} - 3p^{3k} + 3p^{2k} - p^k.$$

Proof Based on Lemma 1, which states that $\Gamma_{\mathbb{Z}_n}$ is a complete graph, the number of vertices is n where $n = p^k$, $k \in \mathbb{N}$ and p is a prime number. Each vertex has degree $p^k - 1$. Therefore, the Forgotten index of the power graph of the group of integers modulo n is given by

$$\begin{aligned} F(G\Gamma_{\mathbb{Z}_n}) &= \sum_{v \in V(\Gamma_{\mathbb{Z}_n})} \text{deg}(v)^3 \\ &= \frac{p^k!}{(p^k - 1)! 1!} (p^k - 1)^3 \\ &= \frac{p^k (p^k - 1)!}{(p^k - 1)! 1!} (p^k - 1)^3 \\ &= p^k (p^{3k} - 3p^{2k} + 3p^k - 1) \\ &= p^{4k} - 3p^{3k} + 3p^{2k} - p^k. \end{aligned}$$

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Conclusion

From the research results above, the ABC index, Nirmala index and forgotten index $ABC(\Gamma_{\mathbb{Z}_n}) = \frac{p^k(p^k-1)}{2} \sqrt{\frac{2p^k-4}{p^{2k}-2p^{k+1}}}$, $N(\Gamma_{\mathbb{Z}_n}) = \frac{p^{2k}-p^k}{2} \sqrt{2p^k-2}$, and $F(G\Gamma_{\mathbb{Z}_n}) = p^{4k} - 3p^{3k} + 3p^{2k} - p^k$ are obtained on the power graph in the modulo integer group for $n = p^k$, where p is a prime number.

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