Dispersion of Solute in Blood Flow of Casson Fluid through Artery with the Effect of Body Acceleration and Temperature

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Abstract
The purpose of this study to investigate the effect of body acceleration and temperature on the solute dispersion in blood flow through an artery treating the blood as a Casson fluid model. The Casson fluid model in a circular straight pipe is formulated mathematically. Velocity of Casson fluid model is determined by solving momentum and constitutive equations. The concentration of solute, dispersion function and mean concentration are obtained by solving the convective diffusion equation using Generalized Dispersion Model (GDM). The influenced of body acceleration and temperature are investigated on the solute dispersion in blood flow and discussed graphically. The velocity of blood flow increases with the increases of body acceleration and slip velocity. When the temperature, yield stress and plug core region increases, the velocity decreases. The body acceleration, yield stress and plug core region increases in the center of the artery and dispersion decreases in the outer region near the wall. Dispersion begins to increase in the outer region near the wall due to the decreases in viscosity causing the temperature in the middle of the artery decreases. Mean concentration increases as the body acceleration, temperature, yield stress, plug core region and stenosis height increases. Acceleration in human body affect the mean concentration of solute. The effects of small temperature variations can cause severe damage to the human body which affect the overall blood behavior in the circulation and reduces viscosity. This research is significant to determine the effectiveness of solute dispersion when body acceleration and temperature are taken into account in blood flow through artery.

Keywords: Temperature; Body acceleration; Solute dispersion; Casson fluid; Generalized Dispersion Model

1. Introduction
In recent development, Toghraie et al. [1] stated that many authors explored area in blood flow since medical science is not able to solve all problems individually. Tiwari et al. [2] analysed the effect of temperature in blood flow of Hershel-Bulkley (H-B) for drug delivery or transportation of nutrient to body that involved solute dispersion model using Generalized Dispersion Model (GDM). Meanwhile, Roy and Beg [3] investigated the temperature-dependent viscosity in blood flow and heat transfer characteristics in an artery with stenosis using implicit finite difference method formula.

Sinha et al. [4] stated that in the human circulatory system, blood flows through different branches of arteries whose heat was distributed through the tissues to the rest of the body where the heat cannot accumulated in any part of the tissue medium. Thus, the temperature was generally stronger depending on blood flow where blood circulation in human body plays an important role in transmission heat between living tissues, especially peripheral veins [1]. Changing on temperature can help to predict the cancers and Siegel et al. [5] stated that temperature has been used to analysing breast cancer. Nowadays, due to unhealthy diet, smoking, diabetes and hypertension may led to cancer. According to World Health Organization that cardiovascular disease (CVD) was leading cause death globally. Haghighi and Chalak [6] stated every day, human body had been exposed to the body acceleration that cause vibration when driving vehicle or rapid body movements in the sports that may led to headaches and increasing heart rate. Jaafar et al. [7] investigated the steady dispersion in blood flow through narrow arteries in chemical reaction using Casson fluid has been solved by using classical
To authors’ knowledge, the composition of variable temperature-dependent viscosity in Casson fluid for dispersion of solutes in blood flow through artery did not receive sufficient attention. Thus, an attempt is made in this study to extend the studies of stenosis through body acceleration by incorporating the effect of temperature in blood flow. Hence, the purpose of this study is to investigate the steady flow behaviour of blood in stenosed artery with the effect of temperature and body acceleration using Casson fluid model. The objectives of this study are to formulate the mathematical model of Casson fluid model through blood flow, to solve the momentum and continuity equation of velocity in blood flow, to solve unsteady convective-diffusion to obtain concentration of solute, dispersion function, longitudinal coefficient and concentration using GDM and to analyse the effect of temperature and body acceleration on the dispersion of solute in blood flow.

2. Mathematical Formulation

2.1. Non-dimensional variables

The following is non-dimensional variables:

\[ r = \frac{r}{R_0}, \quad \tau = \frac{\tau}{\mu_0}, \quad F(t) = \frac{F(t)}{A}, \quad \bar{P} = \frac{P}{\mu_0}, \quad z = \frac{z}{u_0}, \quad u = \frac{u}{u_0}, \quad \tau_s = \frac{\tau_s}{\mu_0}, \quad u_s = \frac{u_s}{u_0}, \quad R(z) = \frac{R(z)}{R_0}, \quad \lambda = \frac{\lambda_0}{\lambda_0} \quad (1) \]

where \( \mu_0 \) is the fluid characteristic velocity, \( r, R_0, \tau, \mu_0, u_0, F(t), \lambda_0, P, z, \tau_s, u_s, R(z) \) are plug core radius, radius of artery in outer region of stenosis, shear stress, viscosity coefficient of Casson fluid, fluid characteristic of velocity, heartbeat movement, circular frequency, radial direction, axial distance, velocity, yield stress, slip velocity and stenosed radius in non-dimensional variables.

2.2. Governing equation

2.2.1. Momentum equation

The non-dimensional momentum equation with body acceleration of steady flow is defined as

\[ B \frac{d \tau r}{dr} = r \left[ \frac{a_0}{\lambda_0} F(t) - B \frac{dp}{dz} \right], \quad (2) \]

where \( a_0 \) is constant parameters of the pressure gradient, \( \lambda_0 \) is the non-dimensional of body acceleration, \( p \) is the pressure, \( \tau \) is the shear stress, \( z \) is the axial coordinate for a circular pipe, \( r \) is the radial coordinate and

\[ B = \frac{\mu(T)u_0}{R_0^2}, \quad (3) \]

where

\[ \mu(T) = \mu e^{-\alpha \left( \frac{\tau - \tau_s}{\tau - \tau_s} \right)}, \quad (4) \]

\( \alpha (\geq 1) \) is the non-dimensional of viscosity parameter index, \( T_w \) is the non-dimensional of temperature of the vessel wall, \( \mu \) is the non-dimensional of viscosity, \( T_e \) is the non-dimensional of equilibrium temperature, according to Tiwari et al. [2] \( \mu(T) \) is the non-dimensional of temperature-dependent viscosity of Reynold model and

\[ F(t) = A_0 \cos(\omega t + \phi) \quad (5) \]

is the non-dimensional of acceleration of the body, \( A_0 \) is denoted as non-dimensional of body acceleration, \( \omega \) is non-dimensional of circular frequency, \( t \) is the non-dimensional of time and \( \phi \) is the
non-dimensional of lead angle of \( F(t) \) which relates to the heartbeat movement. The boundary condition of momentum equation in Eq. (2) is given as follows:

\[
\tau = \text{finite at } r = 0.
\] (6)

### 2.2.2. Convective diffusion coefficient

The non-dimensional of constitutive equation of Casson fluid is given by

\[
-\frac{du}{dr} = \tau + \tau_y - 2\sqrt{\tau} \sqrt{\tau_y},
\] (7)

where \( u \) is the non-dimensional of velocity of Casson fluid, \( \nu \) is the non-dimensional of viscosity coefficient of Casson fluid model and \( \tau_y \) is the non-dimensional of yield stress. The boundary condition is given as follows:

\[
u = \nu_s \text{ at } r = R(z)
\] (8)

where

\[
\tilde{R}(z) = \left. R(z) \right|_{R_0} \left(1 - \frac{\delta}{R_0} \exp \left(\frac{-k^2e^{2z^2}}{R_0^2} \right) \right),
\] (9)

where \( \tilde{R}(z) \) is the radius of the stenosed segment. The non-dimensional of the radius artery in Eq. (9) is given as follows:

\[
R(z) = 1 - \delta e^{(-x^2)},
\] (10)

where \( C = bR_s^{-1} \). \( R_0 \) represents the non-dimensional of radius of the artery in outer region of the stenosis, \( R(z) \) is the non-dimensional of radius of the stenosed segment and \( \delta \) is the non-dimensional of height of stenosis at middle point.

### 2.3. Method of solution

The non-dimensional of velocity expression in the outer non-plug core is given as

\[
u(r) = \nu_s - \frac{1}{2B} \left[ \frac{a_s}{A_h} F(t) - B \frac{dp}{dz} \right] \left[ R^2(z) - r^2 + 2R(z)(r - r_p) \right] - \frac{8}{3} \sqrt{R^2(z)} R_p \left( R(z) - r_p^3 \right),
\] (11)

where \( dp/\!dz \) is the non-dimensional of axial pressure gradient, \( r \) is plug core region, \( r_p \) is the non-dimensional plug flow region, \( \nu_s \) is the non-dimensional of slip velocity and \( R(s) \) is the non-dimensional of radius of the stenosed segment. By evaluating \( r = r_p \) in the Eq. (11), the non-dimensional of velocity of fluid in the plug flow region is given as follows:

\[
u(r_p) = \nu_s - \frac{1}{2B} \left[ \frac{a_s}{A_h} F(t) - B \frac{dp}{dz} \right] \left[ R^2(z) - r_p^2 + 2R(z)(r_p - r_p) \right] - \frac{8}{3} \sqrt{R^2(z)} R_p \left( R(z) - r_p^3 \right),
\] (12)

The non-dimensional of mean velocity is given by

\[
u_m = \int_0^{2\pi} \int_0^\infty \frac{u r dr d\theta}{r dr d\theta}
\] (13)

and has been solved using integral method. It forms
Generalized Dispersion Model (GDM) is a derivative series expansion the approach of Gill and Sankarasubramanian [13] which given as

$$
\frac{\partial C}{\partial t}(z, t) = \sum_{i=1} K_i(t) \frac{\partial C}{\partial z}(z, t).
$$

where $K_i(t)$ is the transport coefficient. The dispersion function of $f_i(r,t)$ plays an important role in calculating the mean concentration, $C_m(z,t)$. The dispersion function is given as follows:

$$
f_i(r,t) = f_{is}(r) + f_{iu}(r,t),
$$

where $f_{is}(r)$ is the dispersion function in the steady state and $f_{iu}(r,t)$ is the dispersion function in the unsteady state that describes the time dependent nature of the dispersion of the solute. The dispersion function at steady state is given by

$$
1 \frac{\partial}{\partial r} \left( r \frac{\partial f_{is}}{\partial r} \right) - \left( u(r) - u_m \right) = 0 \text{ if } 0 \leq r \leq r_p
$$

and the dispersion function in outer region is given as follows:

$$
1 \frac{\partial}{\partial r} \left( r \frac{\partial f_{is}}{\partial r} \right) - \left( u(r) - u_m \right) = 0 \text{ if } r_p \leq r \leq R(z).
$$

Eq. (17) and Eq. (18) are solved using Eq. (18) to get $f_{is}$ and $f_{iu}$,

$$
\frac{df_{is}}{dt}(0) = 0.
$$

and

$$
\frac{df_{is}}{dr}(R(z)) = 0.
$$

The steady dispersion function in the plug flow region, $f_{is}$, and outer flow region, $f_{iw}$, is given as follows:

$$
f_{is} = CI \frac{Ar^2 r_p^2}{48} - \frac{Ar^2 r_p^4}{672R^2(z)} + \frac{1}{12} Ar^2 r_p R(z) - \frac{2}{21} Ar^2 \sqrt{r_p R^2(z)} + \frac{1}{32} Ar^2 R^2(z)
$$

and

$$
f_{iw} = CI \frac{Ar^4}{64} + \frac{8}{147} \frac{Ar^2}{\sqrt{r_p}} - \frac{115}{18} Ar^4 - \frac{1}{18} \frac{Ar^4}{28224} + \frac{Ar^4}{672R^2(z)} + \frac{1}{12} Ar^2 r_p R(z) - \frac{2}{21} Ar^2 \sqrt{r_p R^2(z)} + \frac{1}{32} Ar^2 R^2(z) - \frac{1}{336} Ar^4 \log(r_p) + \frac{1}{336} Ar^4 \log(r_p),
$$

where $A = \frac{1}{(B + F)}$, $P = \frac{dp}{dz}$ and
\[ CI = A \left( \frac{13r_p^4}{7056} + \frac{r_p^5}{5280R^2(z)} - \frac{7r_p^2R^2(z)}{360} + \frac{15r_pR^2(z)}{539} - \frac{R^4(z)}{96} - \frac{r_p^4}{336}\log(r_p) + \frac{1}{336}r_p^4\log(R(z)) \right) \]  

(23)

The general solution of \( f_i(r,t) \) is given as

\[ f_i(r,t) = \sum_{m=1}^{\infty} A_m e^{-\lambda_m J_0(\lambda_m r)} \]

(24)

where

\[ A_m = -\frac{2}{J_0(\lambda_m)} \int_0^z J_0(\lambda_m r) f_i(r) rdr. \]

(25)

The mean concentration is obtained using Inverse Fourier Transform (IFT) [8]. It is given as follows:

\[ C_m(z,t) = \frac{1}{2} \left[ \text{erf} \left( \frac{z - \zeta}{2\sqrt{t}} \right) + \text{erf} \left( \frac{z + \zeta}{2\sqrt{t}} \right) \right]. \]

(26)

From Eq. (26), the local concentration is determined [10]. It is given as follows:

\[ C(r,z,t) = C_m(z,t) + f_i(r,t) \frac{\partial C_m}{\partial z}(z,t). \]

(27)

3. Results and Discussion

3.1. Velocity of the blood flow

The effect of temperature and body acceleration are graphically computed in this section. The results of velocity are obtained and discussed by fixing various parameters in the flow analytic expression after solving the momentum equation and defining the yield stress.

The velocity of Casson fluid with the effect of body acceleration and temperature as illustrated in Figure 1 and the result has been validated with Dash et al. [9]. The result for Casson fluid without the effect of body acceleration shows a good result with the Casson fluid’s velocity in the previous study [9]. Compared to the current result, the results show that increasing of body acceleration tends to increase the velocity of the blood flow.

![Figure 1: Validation of present velocity with Dash et al. [9]](image)

Figure 2 shows the variation of velocity, \( u \) for different values of body acceleration, \( A_i \) in the blood flow with \( \omega = 1, \; t = 1, \; \phi = 0, \; a = 0.02, \; b = 2.5, \; z = 0.5, \; u_i = 0, \; r_s = 0.1, \; B = 2.5 \) and \( P = 2 \). The velocity increases when body acceleration increases. Sudden changing of velocity in human body makes blood flow disrupted.
Figure 3 shows the variation of velocity, \( u \) for different values of temperature, \( T \) in the blood flow with \( A_i = 2, \quad \omega = 1, \quad t = 1, \quad \phi = 0, \quad a = 0.02, \quad b = 2.5, \quad z = 0.5, \quad \tau_y = 0.1, \quad u_r = 0, \quad u_0 = 1, \quad R_o = 1 \) and \( P = 2 \). The velocity decreases when temperature increases. Due to fluid viscosity, there is loss of kinetic energy of fluid to heat, which tend to increase fluid temperature, but in ordinary situations this heating is negligible. Changes in temperature can change the viscosity of the blood.

3.2. Steady dispersion function
The steady dispersion of Casson fluid with the effect of body acceleration and temperature as illustrated in Figure 4 and the result has been validated with Dash \textit{et al.} \cite{9}. Without the impact of body acceleration, the result of Casson fluid’s steady dispersion shows a good result in the present study \cite{9}. The present results show the steady dispersion decreases in outer region near the wall, increases of body acceleration at the centre of artery compared to the current results.

Figure 4: Validation of present steady dispersion with Dash \textit{et al.} \cite{9}

Figure 5 shows the variation of steady dispersion function for different values of body acceleration, \( A_i \) in the blood flow with \( \omega = 1, \quad t = 1, \quad \phi = 0, \quad a = 0.02, \quad b = 2.5, \quad z = 0.5, \quad u_r = 0, \quad \tau_y = 0.1, \quad B = 2.5 \) and \( P = 2 \). When body acceleration increases at the centre of artery and steady dispersion decreases in outer region near wall. As the amplitude of the body’s acceleration increases, blood flow decreases, causing the dispersion function to decrease and the dispersion of solutes to be affected by fluctuating blood flow.

Figure 6 shows the variation of steady dispersion function for different values of temperature, \( T \) in the blood flow with \( A_i = 0.1, \quad \omega = 1, \quad t = 2.8, \quad \phi = 0.01, \quad a = 0.01, \quad b = 0, \quad z = 0.05, \quad B = 2.5, \quad \tau_y = 0.01, \quad P = 2, \quad u_0 = 1 \) and \( R_o = 1 \). When temperature decreases at the centre of artery and steady dispersion increases in outer region near wall. An increase in varying viscosity parameters leads to growth in axial scattering due to decreased viscosity and smoother flow velocity.
3.3. Unsteady dispersion function

Figure 7 shows the variation of unsteady dispersion function for different values of body acceleration, $A_0$, in the blood flow with $\omega = 1$, $t = 0$, $\phi = 0.05$, $a = 0.01$, $b = 0$, $z = 0.5$, $B = 1$, $\tau_s = 0.01$ and $P = 1$. The dispersion function increases as the amplitude of body acceleration increases at the center of artery and decreases in the outer region near the wall. As the amplitude of the body’s acceleration increases, blood flow decreases, causing the dispersion function to decrease and the dispersion of solutes to be affected by fluctuating blood flow.

Figure 8 shows the variation of unsteady dispersion function for different values of temperature, $T$, in the blood flow with $A_0 = 0.1$, $\omega = 1$, $t = 0$, $\phi = 0.05$, $a = 0.01$, $b = 0$, $z = 0.5$, $B = 1$, $\tau_s = 0.01$, $P = 1$, $u_0 = 1$ and $R_0 = 1$. Temperature increases, the unsteady dispersion function decreases. The heat transfer aspect significantly affects the axial propagation in the flow through tubes with medium or less reactive walls.

3.4. Dispersion function

Figure 9 shows the variation of dispersion function for different values of body acceleration, $A_0$, in the blood flow with $\omega = 1$, $t = 0.2$, $\phi = 0.02$, $a = 0.01$, $b = 0$, $z = 0.5$, $B = 2.5$, $\tau_s = 0.01$ and $P = 3$. It is showed that dispersion function decreases with the body acceleration and it approaches when the
value of body acceleration is increasing. According to Patel and Sirs [10], the flexing of red blood cells and their migration of the core is one of the factors that influence the dispersion of solute in blood flow.

Figure 10 shows the variation of dispersion function for different values of temperature, $T$ in the blood flow with $A_e = 1$, $\omega = 1$, $t = 0.2$, $\phi = 0.02$, $a = 0.01$, $b = 0$, $z = 0.5$, $B = 2.5$, $\tau_s = 0.01$, $P = 3$, $u_0 = 1$ and $R_0 = 1$. The dispersion function decreases with the increasing of temperature. According to Tiwari et al. [2], the varying viscosity properties lead to a reduction in viscosity and result in a smoother flow in the region.

3.5. *Mean concentration*

Figure 11 shows the variation of mean concentration for different values of body acceleration, $A_e$ in the blood flow with $F(t) = 2$, $a = 0.02$, $b = 0$, $z = 5$, $B = 1$, $\tau_s = 0.75$, $z_s = 1.4$, $u_0 = 11$, $Pe = 1$, $R_0 = 2.5$ and $P = 1.5$. The mean concentration of solute increases as the pressure gradient increases. Acceleration in the body, then a sharp decrease in mean concentration. The heart pumps blood with high flow when the concentration of solute increases.

Figure 12 shows the variation of mean concentration for different values of temperature, $T$ in the blood flow with $F(t) = 2$, $a = 0.02$, $b = 0$, $z = 5$, $A = 1$, $\tau_s = 0.75$, $z_s = 1.4$, $u_0 = 11$, $Pe = 1$, $R_0 = 2.5$ and $P = 1.5$. Temperature increases, the mean concentration increases. The temperature-dependent viscosity reduces the difference in mean concentration. The effects of small temperature variations can cause severe damage to the human body.
Figure 12: Variation of mean concentration for different values of temperature, $T$ in the blood flow with $F(t) = 2$, $a = 0.02$, $b = 0$, $z = 5$, $A = 1$, $\tau_s = 0.75$, $z_s = 1.4$, $\alpha_s = 11$, $Pe = 1$, $R_0 = 2.5$ and $P = 1.5$

4. Conclusion

The results indicate the higher of temperature, the lower the flow of velocity. Then, the velocity of blood flow increases in blood flow while velocity increases with the increases of pressure gradient. Meanwhile, temperature decreases at the centre of artery and steady dispersion increases in outer region near wall. Then, body acceleration increases at the centre of artery and steady dispersion decreases in outer region near wall. Besides, temperature increases, the unsteady dispersion function increases. Then, when body acceleration increases at the centre of artery and steady dispersion decreases in outer region near wall. The dispersion function increases and it approaches zero when temperature increases. Meanwhile, the dispersion function decreases and it approaches when the value of body acceleration increases. For mean concentration, temperature increases, the mean concentration increases. The temperature-dependent viscosity reduces the difference in mean concentration. The mean concentration of solute increases as the pressure gradient increases. Acceleration in the body, then a sharp decrease in mean concentration. The current findings are helpful in addressing the issues of dispersion in human body. In the future, this research can be extended to two different models. It should also be noted that the velocity and flow rate of the two-fluid blood flow model are higher than single fluid blood flow model.

References


