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Forecasting the Earthquake Events in West Nusa Tenggara using Poisson Hidden Markov Model

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Abstract
Province of West Nusa Tenggara (NTB) is one of Province in Indonesia which is often hit by earthquakes that causes horrible impacts on the surrounding community. One way to reduce the impacts is by predicting the number of earthquakes in the future by the Poisson Hidden Markov Model (PHMM). PHMM is a stochastic process, which involves hidden states or latent variables and observed variables. The hidden state in this study is seismicity levels, which includes three states (low-level, medium-level, and high-level). PHMM can be used in spite of the existing of over-dispersion and dependency relationship among the data. This research used earthquake event data that consists of (a) *in sample data* (January 2009 - September 2018) and (b) *out of sample data* (October 2018 – April 2019), achieved from UGSG (United States Geological Survey). The earthquake magnitude used in this study is ≤ M4.7 and the depth is < 60 km. The parameter estimation method used in this study is the Bayesian method. This study aims (a) to predict the number of earthquake events in NTB Province over the next 10 months and (b) to observe the accuracy of forecasting results based on MAE values. The study shows that the forecasting results of earthquake events in NTB in October 2018 is 2.0131, November 2018 is 1.7927, December 2018 is 1.7977, January 2019 is 1.8236, February 2019 is 1.8446, March 2019 is 1.8425, April 2019 is 1.8307, May 2019 is 1.8162, June 2019 is 1.8219 and July 2019 is 1.8219. In other words, the results of forecasting the number of earthquake events in October 2018 to July 2019 ≈ 2 events. The accuracy of the forecasting results for the next 10 months is moderate with MAE values of 1.0949.
Keywords: Earthquake; Poisson Hidden Markov Model; Bayesian; MAE

1. Introduction

West Nusa Tenggara (NTB) is one of the provinces in Indonesia that was often hit by the earthquakes. It has a big impact on the people around the area, such as death and injured victims, house damage and public facilities, and materials losses. Therefore, in order to reduce the negative impact of earthquakes is by forecasting the number of earthquake events in the future.

The analysis method that can be used to predict the number of earthquake events is by using Poisson Hidden Markov Model (PHMM) analysis. PHMM is applicable for count time series data which have over-dispersed (when the variance is greater than average) and dependency relationships [1].

PHMM analysis is the development of a Markov model that consists of hidden states and observed states. Hidden state is a variable that cannot be observed directly or latent variable. The hidden state used in this study is the seismic activity level, assuming that the level of seismic activity causing an earthquake cannot be observed directly and forms a Markov chain. The hidden state in this research is classified into three levels: the low level, the medium level and the high level. While the observed state applied in this research is the number of earthquakes cases (magnitude ≥ M4.7 and depth < 60 km) in NTB from January 2009 to April 2019, obtained from USGS (United States of Geological Survey).

A lot of PHMM application for earthquake cases was carried out in the previous studies, such as HMM using Generative Embedding method [2], the Multivariate PHMM method using the EM (Expectation-Maximization) algorithm [3], PHMM through the EM Algorithm [4, 5], Poisson Process (PP) and PHMM methods through the EM Algorithm [6 Can, 2014], HMM using Baum Welch method [7], HMM method for earthquake declustering by Viterbi Algorithm [8]. Based on those researches, it shows that parameter estimation in the previous studies mostly uses the Maximum Likelihood.

The other previous research in earthquake cases uses 2D-Hidden Markov Model (2D-HMM) using Markov Chain Monte Carlo (MCMC) [9]. That research used categorical variables such as geological surface and more related to spatial fields. However, this method is complex and not suitable for count time series data. While this research is using the PHMM method which is more appropriate for count time series data. This research is the development of previous research [10]. The research observed the best parameters using the PHMM two-state method, which is simpler, of course, but excludes the forecasting method. While this research proposes PHMM three-states and forecasts the number of earthquake events in the next time. Information about earthquake forecasting can be used as a reference in earthquake mitigation strategies or applied in other fields.

This research aims to (1) predict the number of earthquakes in the next 10 months using PHMM analysis with Bayesian approach and (2) find out the accuracy of the forecasting results based on the MAE value. The amount of MAE value is obtained by comparing the forecasting results of earthquake events and *out of sample data* in October 2018-April 2019. The smaller the MAE value, the better the forecasting results will be.

2. Literature Review

2.1. Bayesian

2.1.1. Bayesian Concept

The Bayesian theorem [11] is defined as equation (2.1):

$$f(\beta|x) = \frac{f(x\mid\beta)f(\beta)}{f(x)} \propto f(x\,|\beta)f(\beta) \qquad (2.1)$$

where:

$f(\beta) = prior\ distribution$

$f(x|\beta)$ = likelihood function, is the cumulative density function from *n* variables $X_1, X_2, X_3, \dots X_n$, expressed in terms of $P(x_1, x_2, \dots x_n|\beta)$ in equation (2.2).

$$f(x|\beta) = \prod_{i=1}^{n} f(x_i\,|\beta) = f(x_1|\beta)f(x_2|\beta)\dots f(x_n|\beta) \qquad (2.2)$$

$f(\beta|x)$ = *posterior distribution*

2.1.2. Markov Chain Monte Carlo

Posterior distribution in the Bayesian method is difficult to be analyzed numerically because of its complex calculations. The method that can be done to overcome this problem is the simulation method. MCMC (Markov Chain Monte Carlo) is the simulation method that is very flexible in predicting parameters related to Bayesian analysis [12]. One of the most applied MCMC is Gibbs Sampler.

The Markov chain is a stochastic process [11]:

$\beta\ = \{\beta^{(1)}, \beta^{(2)}, \dots, \beta^{(G)}\}$,

where: $f\left(\beta^{(g+1)}|\beta^{(g)}, \dots, \beta^{(1)}\right) = f\left(\beta^{(g+1)}|\beta^{(g)}\right)$.

The step in the MCMC produce random samples

$\beta_q = \beta^{(1)}, \beta^{(2)}, \dots, \beta^{(G')}$

where:

G'=G-V.

G = the number of generated iterations

V = number of *burn in.*

The estimated parameters of posterior distribution can be acquired in every function of $F(\beta)$ in parameters β by the equation (2.3):

$$\hat{E}(F(\beta\,)|x) = \overline{F(\beta\,)} = \frac{\sum_{g=1}^{G'} F(\beta^{\,(g)})}{G'} \qquad (2.3)$$

The *standard deviation* values can also be achieved by the equation (2.4).

$$\widehat{SD}\;(F(\beta)\mid x) = \frac{\sum_{g=1}^{T'} \left[F(\beta^{(g)}) - \hat{E}(F(\beta\,)\mid x)\right]^2}{G'-1} \qquad (2.4)$$

2.1.3. Convergence Testing of Estimated Parameters

The convergence testing of Bayesian estimated parameters can be checked based on the autocorrelation function plot (ACF plot) [11]. The ACF plot reflects the autocorrelation value among simulated parameter estimation.

The estimated parameter is decided to be convergence when the lag 1 in ACF plot is significance, while the other lag (> lag 1) is not significance and the value is close to zero as the lag increases. A lag is also said to be significant [13] when it exceeds the upper and lower limits of the autocorrelation value of 1.96 /$\sqrt{n}$, where the value of 1.96 is quantile 0.975 on normal distribution with α = 0.05.

If the estimated parameter is not convergence, the number of iteration can be increased or thinning can be applied. The selection of thinning is done by monitoring the ACF plot and choosing the lag sampling > 1 which have low autocorrelation value. The optimal thinning selection is choosing based on the ACF plot which provides the most convergence of estimated parameters.

2.1.4. Checking of Estimated Parameter Accuracy

The accuracy of Bayesian estimated parameters can be detected based on MCSE or Monte Carlo Standard Error value [11]. The MCSE shows the amount of variance in each estimated parameter obtained during the simulation. [14] If the value of MCSE < 5% × standard deviation, it can be concluded that the estimated parameters are accurate.

2.1.5. Significance Testing of Estimated Parameters

The significance of Bayesian estimated parameters can be tested using 95% credible intervals through the lower limit of the quantile 0.025 and the upper limit of the quantile 0.975 [11]. The estimated parameter is concluded to be significant when the range in the quantile does not contain a zero value.

2.2. Poisson Hidden Markov Model Using Bayesian Method

Poisson Hidden Markov Model is a bivariate stochastic process (M and X) on discrete time series data that consists of M states and sequential observations on X stochastic processes (observed variable) which has a non-negative-integer value [6]. The distribution of X observations (state-dependent process) is generated from the Poisson distribution.

The average and variance values in Poisson distribution [15] can be counted as the equation (2.5) and equation (2.6):

$$E(X) = \lambda \qquad (2.5)$$
$$Var\;(X) = E(X^2) - E(X) = \lambda \qquad (2.6)$$

The data is over-dispersed when $Var(X)$ > $E(X)$. As a result, the data can be analyzed using PHMM. The characteristics found in PHMM [1, 16, 17]:

a) Observations that can be observed directly defined as observed variable, where $X = \{X_1, X_2, X_3, \ldots, X_T\}$.
b) Symbol *d* as the number of hidden state.

The *d* state set of PHMM expressed as $M = \{\lambda_1, \lambda_1, \ldots, \lambda_d\}$, where M_t is illustrated as state at the time interval t.

c) Initial state distribution or $\theta(1) = P(M_1 = i) = \theta(i)$, where $\theta_i \in \Theta$ and $1 \le i \le d$.
d) Symbol ***S*** described as the probability for transition from state λ_i at time *t* to λ_j at time $t+1$ with order $d \times d$, where $S_{ij}(t) = \{s_{ij}(t)\}$.
e)

$$s_{ij}(t) = P(M_{t+1} = \lambda_j | M_t = \lambda_i) \quad (2.7)$$

for $i, j = 1, \ldots, d$; at time $t = 1, 2, \ldots T$.
The matrix ***S*** in Markov Chain is said to have stationary distribution $\boldsymbol{\theta}$, where

$$\boldsymbol{\theta S} = \boldsymbol{\theta} \text{ and } \boldsymbol{\theta 1'} = \mathbf{1} \quad (2.8)$$

The important performance of ***S*** in PHMM is following the homogeneity rule that fulfill the *Chapman Kolmogorov:*
$\boldsymbol{S}(\boldsymbol{t}) = \boldsymbol{S}(\mathbf{1})^{\boldsymbol{t}}$, for $t \in T$
The number of transition probability in each row is 1.

f) State dependent probability distribution or $\phi_i(y)$ illustrated with matrix $\boldsymbol{\phi}$ which consists of element $\phi_i(y)$.

(2.9)

$$\phi_i(y) = P(X_t = x | M_t = \lambda_i) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & \text{, for } x = 1, 2, 3, \ldots \\ 0 & \text{, others} \end{cases}$$

g) The marginal probability in PHMM, expressed by the equation (2.10):

$$P(X_t = x) = \sum_{i=1}^{d} P(M_t = i) P(X_t = x | M_t = \lambda_i) = \sum_{i=1}^{d} \theta_i(t) \phi_i(x) \quad (2.10)$$

The parameter estimation used in this research is the Bayesian approach. The prior distribution is an important aspect of the Bayesian method because the prior distribution is used as a reference in determining the posterior distribution. This study used a conjugate prior distribution because the prior distribution and the posterior distribution come from one distribution family [11]. In the PHMM method, there are two parameters in PHMM analysis, namely parameters $\boldsymbol{\lambda}$ and $\boldsymbol{\Gamma}$.

The parameter λ_j cannot be calculated directly but is calculated using the parameter τ_j. The parameter λ_j has a Poisson distribution while the parameter τ_j has a Gamma distribution. The Gamma distribution with parameters a and b is prior conjugate for data with Poisson distribution with parameter $\boldsymbol{\lambda}$ [11]. In other words, the Gamma distribution is a family of the Poisson distribution. Therefore, the prior distribution used in τ_j has a Gamma distribution.

While the prior distribution $\boldsymbol{\Gamma}$ has the Dirichlet distribution, where the elements of the matrix $\boldsymbol{\Gamma}$, namely γ_{ij} with *i, j* = 1, 2,…, *m*. This distribution is determined by considering that γ_{ij} is the probability of transition from state *i* to *j*, which is between zero and one and the number of matrix elements in each row is one. Dirichlet values are between zero and one and the total number of probability is one [18]. This is suitable for the characteristics of parameters $\boldsymbol{\Gamma}$ in the PHMM analysis. As a result, the prior distribution used for the parameter $\boldsymbol{\Gamma}$ in the PHMM analysis uses the Dirichlet distribution.

2.3. Forecasting using PHMM Analysis

Forecasting the number of earthquakes in the next 10 months is conducted by using the expectation value [19]. The forecasting in October is conducted based on the results of the parameter estimation, obtained from *in sample data*, while the forecasting in the next month is applied based on both of *in sample data* and the forecasting results data in the previous period.

2.4. Accuracy of the Forecasting Results

The accuracy of forecasting results can be observed based on the achieved of MAE value [20]. For example, the amount of error in forecasting in the time period *t*, is denoted by e_t. The error value is the difference between the actual data or *out of sample data* (x_t) and the forecasting result data at time *t* $(\hat{x}_t)$.

3. Methodology

3.1. Research Data

The data in this study is the number of earthquakes occurring in NTB from January 2009 to April 2019 with magnitude scale ≥ M4.7 at depth < 60 km, obtained from USGS (United States Geological Survey). The data in this study is classified into *in sample data* and *out of sample data*. Data in January 2009-September 2018 is chosen as *in sample data*. This data will be used to estimate three-state PHMM parameters, while the *out of sample data* is the actual data in October 2018 – April 2019. The *in sample data* and *out of sample data* will be compared to evaluate the accuracy of forecasting results by MAE.

3.2. PHMM Analysis using Bayesian Approach

The stages in the analysis of PHMM using Bayesian approach are as follows:

1) Inputing the research data.
2) Over-dispersion checking on the research data using equation (2.5) and equation (2.6).
3) Analyzing the PHMM using the following Bayesian simulation method [21, 22, 23]:
   a) Determining the following functions:
   Likelihood function:

$$f(x,m \mid \hat{\lambda},\hat{S}) \propto \hat{\theta}_o \left(\prod_{i=1}^{d}\prod_{j=1}^{d}\left(\hat{s}_{ij}\right)^{n_{ij}}\right) \times \left(\prod_{j=1}^{d}\hat{\lambda}_j^{\sum_t x_{ct}} \exp\left(-\hat{n}_j\hat{\lambda}_j\right)\right) \quad (2.11)$$

$$\hat{S} \sim Dirichlet\ (\hat{\zeta}_r) \quad (2.12)$$

$$\hat{\eta}_j \sim Gamma\ (\hat{q}_j, \hat{o}_j) \quad (2.13)$$

$$\hat{\lambda}_j = \sum_{j=1}^{i}\hat{\eta}_j. \quad (2.14)$$

It means that the estimated parameter $\hat{\lambda}_i$ will be obtained once the estimated parameter $\hat{\eta}_j$ is achieved.

   b) Performing initialization for estimated parameters of $\hat{S}$, $\hat{\eta}$ and $\hat{\lambda}$.
   c) Conducting simulation on Hidden State

   The recursion process under the hidden state is as follows:

1. *Forward Step*

   a. $P(m_t = j|X_{t-1},\beta)$ (2.15)

$= \sum_{i=1}^{d} s_{ij} \times P(m_{t-1} = i|X_{t-1},\beta)$

for $j = 1,2,\ldots,d$

   where $P(m_0 = i|X_0,\beta) = 1/d$ is the *inisial state distribution.*

b. Updating step by using equation (2.16).

   $P(m_t = j|X_t,\beta) \propto P(m_t = j\,|X_{t-1},\beta) \times \phi_i(x)$ (2.16)

   Where $\phi_i(x)$ is *state dependent probability* on equation (2.9).

   Repeat the step a and b at $t = 1,2,\ldots,T$.

2. Generating m_T from $P(m_T = j|X_T,\beta)$.
3. *Backward Step*

   For example, the generated $m_{t+1} = w$ and for $j = 1,2,\ldots,d$, then:

   $P(m_t = j|X_T,m_{t+1} = w,\beta\,) \propto s_{jl} \times P(m_t = j|X_t,\beta)$ (2.17)

   Repeat this step at *t = T-1,T-2, …2, w.*

4. Generate m_t from $P(m_t = j|X_T,m_{t+1} = r,\beta)$.

   d) Updating the estimated parameters

$\hat{S}_r \sim Dirichlet(\zeta_r + \hat{n}_r)$ (2.18)

$\hat{\eta}_j \sim Gamma(\hat{q}_j + \sum_{t=1}^{T} x_{jt}, \hat{o}_j + \hat{N}_j)$ (2.19)

While $\hat{\lambda}_j$ can be achieved using equation (2.14)

e) Repeating step (c) and step (d) as much as G iterations until the estimated parameters have convergence based on the ACF plot.

f) The estimated parameters $\hat{S}_r$ and $\hat{\lambda}_j$ will be obtained, which are the averages of the posterior distribution in equation (2.3).

4) Accuracy testing of the estimated parameters Bayesian approach can be done based on MCSE.

This method is conducted by dividing the output of samples in batches as much as G. The MCSE equations is defined in as following equation [11]:

$$MCSE[F(\beta)] = \sqrt{\frac{1}{Q}}\widehat{SD}\left[\overline{F(\beta)_b}\right] \quad (2.20)$$

where:

SD = *standard deviation* based on the equation (2.4)

$\overline{F(\beta)}_b = \frac{1}{c}\sum_{t=(b-1)c+1}^{bc} F(\beta^{(t)})$ for each batch *b*.

b = 1, …, G

5) Significance testing of the estimated parameters Bayesian approach is conducted by credible interval on the quantile 0.025 and quantile 0.975.

6) Once the estimated parameters $\hat{S}$ are obtained, then the stationary state distribution $\hat{\theta}$ is calculated based on equation (2.8).

7) Checking the goodness of fit of PHMM three-state based on DIC on equation (2.21) [24, 25].

$$DIC = \overline{D} + p_D \quad (2.21)$$

where:

$D(\theta) = D(x,\beta) = -2\log p(x \mid \beta)$, is deviance.

$\overline{D} = E(D(\beta))$

$p_D = \frac{1}{2}\widehat{var}(D(x,\beta) \mid x)$

The smaller the DIC value, the better the model will be.

8) Forecasting the number of earthquake events based on the expected value in the equation (2.22) [1].

$$\hat{X}_t = E(X_t) = \sum_{i=1}^{d} P(M_t = i)E(X_t = x \mid M_t = \lambda_i) \quad (2.22)$$

$$\hat{X}_t = \sum_{i=1}^{d} \theta_i \lambda_i = \boldsymbol{\theta}'\boldsymbol{\lambda}$$

9) Evaluating the forecasting result based on MAE value on equation (2.23) [20].

$$MAE = \frac{1}{z}\sum_{t=1}^{z} |e_t| \quad (2.23)$$

where:

$e_t = x_t - \hat{x}_t$

x_t = the actual data or *out of sample data*

$\hat{x}_t$ = forecasting result data at time *t*

z = number of forecasting period.

The research used software R (package *R2OpenBUGS, coda*, *MCMCpack*, *mcmcse* and *Hidden Markov*) and applied the coding of Gibbs Sampler in PHMM [23].

4. Results and discussion

4.1. Over-dispersion Checking

The average number of earthquakes in NTB with a magnitude ≥ M4.7 and the depth of the earthquake < 60 km for the period January 2009 - September 2018, *E(Y),* is 0.6325. While the variance of data, *Var (Y),* is 14.0448. It shows that *Var (Y) > E (Y)*. This means that the data is over-dispersed. As a result, analysis on PHMM can be conducted to the next step.

4.2. Estimated Parameters of PHMM Three-State

4.2.1. Result of Convergence Testing of Estimated Parameters

The number of iterations used PHMM three-state is 200,000 iterations, 5,000 burn in period and 60 thinning. The result of ACF plot for PHMM three-state is displayed in Figure 1.

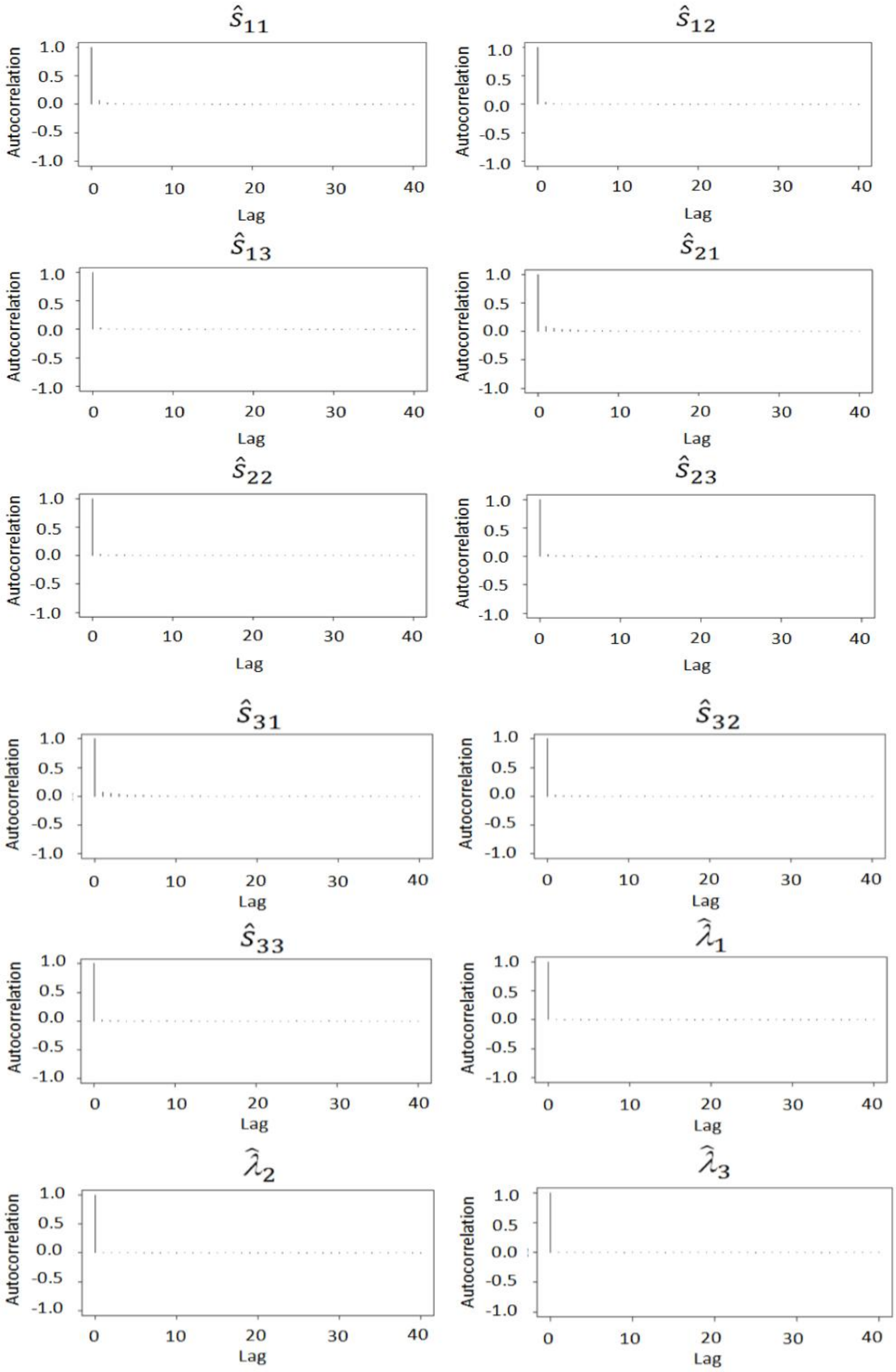

Figure 1 ACF Plot of PHMM Three-State using Bayesian Approach

Based on the ACF plot in Figure 1, the simulation of PHMM three-state generates the convergence estimated parameters. The figure shows that lag 1 is significance, but the others are not; and autocorrelation value of lag > 1 closes to zero values as the lag increases. In addition, the lag > 1 does not seem to exceed the upper and lower limits of the autocorrelation value of $1.96/\sqrt{n}$, that is ± 0.1812. Therefore, it can be concluded that the estimated parameters are convergence or fulfill the targeted posterior distribution.

4.2.2. Accuracy Testing of Estimated Parameters

The results of estimated parameters accuracy using the MCSE and standard deviation values is provided in Table 1.

Based on Table 1, it shows that the MCSE value is < 5% $\times$ standard deviation. Therefore, it can be concluded that the estimated parameters of PHMM three-state using Bayesian approach are accurate.

Table 1: Result of Accuracy Testing of Estimated Parameter

Estimated Parameters	**SD***	**5% x SD**	**MCSE****
$\hat{\lambda}_1$	0.0384	0.0019	8.4240×10^{-5}
$\hat{\lambda}_2$	2.0897	0.1045	4.6727×10^{-3}
$\hat{\lambda}_3$	6.3819	0.3191	1.3477×10^{-2}
$\hat{s}_{11}$	0.0384	0.0019	3.7175×10^{-5}
$\hat{s}_{12}$	2.0902	0.1045	2.9148×10^{-5}
$\hat{s}_{13}$	5.9966	0.2998	1.9352×10^{-5}
$\hat{s}_{21}$	0.0144	0.0007	5.5004×10^{-4}
$\hat{s}_{22}$	0.0119	0.0006	4.6832×10^{-4}
$\hat{s}_{23}$	0.0082	0.0004	5.5549×10^{-4}
$\hat{s}_{31}$	0.1977	0.0099	5.5507×10^{-4}
$\hat{s}_{32}$	0.1905	0.0095	5.5842×10^{-4}
$\hat{s}_{33}$	0.2226	0.0111	4.4668×10^{-4}

**) SD: Standard Deviation; **) MCSE: Monte Carlo Standard Error*

4.2.3. Significance Testing of Estimated Parameters

The significance of the parameters are tested using the credible interval values found in Table 2.

Table 2: Result of Significance Testing of Estimated Parameters

Estimated Parameters	***Mean***	***Credible Interval***	
		0.025	**0.975**
$\hat{\lambda}_1$	0.1664	0.0999	0.2494
$\hat{\lambda}_2$	8.0256	4.3910	12.5300
$\hat{\lambda}_3$	37.0735	26.2300	49.6900
$\hat{s}_{11}$	0.9749	0.9399	0.9949
$\hat{s}_{12}$	0.0168	0.002	0.0468
$\hat{s}_{13}$	0.0082	0.0002	0.0304
$\hat{s}_{21}$	0.2667	0.0104	0.7245
$\hat{s}_{22}$	0.2461	0.0081	0.6958
$\hat{s}_{23}$	0.4872	0.089	0.8983
$\hat{s}_{31}$	0.2606	0.0100	0.7164
$\hat{s}_{32}$	0.4939	0.0935	0.9012
$\hat{s}_{33}$	0.2455	0.0084	0.6969

Based on Table 2, it seems that the range of credible interval value between the quantile of 0.025 to 0.975 does not contain a value of 0. Thus, it can be said that all estimated parameters on PHMM three-state using Bayesian approach are significant.

4.2.4. Estimated Parameters Results of PHMM Three State using Bayesian Approach
The estimated parameters on PHMM three state using Bayesian approach can be summarized in Table 4.3.

Table 3: The Estimated Parameters of PHMM Three-State

Estimated Parameters	**Symbol**	**PHMM Three State**
Hidden State	$\hat{\mathbf{M}}$	$[0.1664 \quad 8.0256 \quad 37.0735]$
Transition probability	$\hat{\mathbf{S}}$	$\begin{bmatrix} 0.9749 & 0.0168 & 0.0082 \\ 0.2667 & 0.2461 & 0.4872 \\ 0.2606 & 0.4939 & 0.2455 \end{bmatrix}$
Stationary distribution	$\hat{\boldsymbol{\theta}}$	$[0.9132 \quad 0.0467 \quad 0.0401]$

Based on Table 3, it can be concluded that the average number of earthquake events caused by the low seismic activity level is almost 0 event. While the earthquake caused by the medium level was 8 events and the high level was 37 events.

If the seismic activity level in the previous period was low, the probability of seismic activity levels being low in the next period would be 58 times bigger than the medium level and 119 times higher than the high level. If the seismic activity level was medium in the previous period, then the probability of seismic activity levels being high in the next period would be 1.8 times higher than the low level and twice bigger than the medium level. If seismic activity level was high in the previous period, the probability of seismic activity levels being medium in the next period would be 1.9 times bigger than the low level and twice higher than the high level.

The number of monthly earthquakes caused by the low seismic activity level in the long run behaviour (steady state) is bigger (107 events out of 117 events) than the medium level (5 events) and the high level (5 events).

Goodness of fit of PHMM three-state based on DIC value is 130.9350. It means that the model is moderate. As a result, the model can be used to forecast the number of earthquake events in the next time.

4.3. Forecasting Results of PHMM using Bayesian
The forecasting results of PHMM three-state using Bayesian approach displayed in Table 4. Based on Table 4, it shows that the rounding data forecasted from numbers of earthquake events in NTB with magnitude ≥ M4.7 and earthquake depth < 60 km, from October 2018 to July 2019, generates the same forecasting result which is equal to two events. The result of forecasting in the first period ($\hat{x}_1$) is the expectation value of the *in sample data* (*X*). Furthermore, *in sample data* and the forecasting data on the first period ($\hat{x}_1$) is used as the input data for getting the expectation value. This value is the second-period forecasting result ($\hat{x}_2$). Then, the forecasting data for the next period ($\hat{x}_z$) is the expectation value of *in sample data* (X) and the previous forecasting data ($\hat{x}_{z-1}, \hat{x}_{z-2}, \dots, \hat{x}_1$). Moreover, the forecasting results in June and July 2019 produces the same results. This indicates that the long-term forecasting for earthquake cases in this study will produce the same results.

The value of MAE is 1.0949. This value is obtained based on the forecasting results of earthquake events compared with *out of sample data* in October 2018 - April 2019. It means that the forecasting results of earthquake events in NTB, with a magnitude scale ≥ M4.7 and the depth of the earthquake < 60, is moderate.

Table 4. The Forecasting Result in October 2018 – July 2019

Period	***Out of Sample Data*** $(\mathbf{x_t})$	**Forecasting Result** $(\hat{\mathbf{x}}_\mathbf{t})$
October 2018	1	2.0131 ≈ 2
November 2018	0	1.7927 ≈ 2
December 2018	3	1.7977 ≈ 2
January 2019	1	1.8236 ≈ 2
February 2019	0	1.8446 ≈ 2
March 2019	2	1.8425 ≈ 2
April 2019	1	1.8307 ≈ 2
May 2019		1.8162 ≈ 2
June 2019	-	1.8219 ≈ 2
July 2019		1.8219 ≈ 2

Conclusion

The forecasting results of earthquake events in NTB in October 2018 is 2.0131, November 2018 is 1.7927, December 2018 is 1.7977, January 2019 is 1.8236, February 2019 is 1.8446, March 2019 is 1.8425, April 2019 is 1.8307, May 2019 is 1.8162, June 2019 is 1.8219 and July 2019 is 1.8219. In other words, the results of forecasting the number of earthquake events in October 2018 to July 2019 ≈ 2 events. The accuracy of the forecasting results for the next 10 months is moderate with MAE values of 1.0949. Based on result of the analysis, actual data looks different with forecasting data. Therefore, another forecasting method is needed for next research so that the obtained forecasting result could describe the actual condition of earthquakes.

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